TIME = MONEY

"An imbalance between rich and poor is the oldest and most fatal ailment of all republics."
Plutarch (46 – 120 A.D.)

A new mathematical approach to macroeconomics that introduces time valuation and time accounting and relates the natural dynamics of economies to the consolidated financial statements of all companies and financial institutions in a closed economy with the purpose to build a sustainable financial system for creation and distribution of wealth over long periods of time by
Daan Koppen

SUPPORTING QUOTES ABOUT THE TIME AND MONEY EQUIVALENCE (PART 1 TIME TO VALUE)

"When you go down to the store and buy this pencil, you are in effect trading a few minutes of your time for a few seconds of the time of all those thousands of people who co-operated to make this pencil." [1]

Milton Friedman (1980), while presenting his vision of how the free market might bring about world peace, uses a pencil to describe how the value of goods is gradually and exclusively created by labour that transfers initially useless raw materials into useful goods. Secondly, he refers to a transaction between the buyers' time and the sellers' time thereby supporting both the concept that time equals money and implying the existence of human time flows in the real economy.

"Time is the coin of your life. It is the only coin you have, and only you can determine how it will be spent." [2]

Carl Sandburg (1963) states that time is the only thing that humans really own in life and stresses the importance of spending it wisely. He uses money as a metaphor.

"Time is money says the proverb but turn it around and you get a precious truth. Money is time."[3]

George Gissing (1903) argues in "The Private Papers of Henry Ryecroft" that if you have money you are free to enjoy a comfortable life, while otherwise your time would have belonged to an employer. "With money I buy for cheerful use the hours which otherwise would not in any sense be mine".

"One today is worth two tomorrows." [4]

Benjamin Franklin (mid-18th century) makes the point that your present time is more valuable than your future time, because your future time is not guaranteed. This is the exact concept of "the time value of time" which in analogy with the time value of money states that present time is more valuable to humans than future time.

SUPPORTING QUOTES ABOUT THE IMPACT OF INEQUALITY (PART2 MONEY TO SHARE)

"…the median income adjusted for inflation of a fulltime male worker in The United States is at the same level that it was 42 years ago…at the bottom things are worse." [5]

Joseph Stiglitz (2018) makes the point that all labour productivity gains of the past decades resulted into increased profitability instead of increased wages.

"As long as poverty, injustice and gross inequality exist in our world, none of us can truly rest."[6]

Nelson Mandela (2005) argues in his Make Poverty History speech that "sometimes it falls upon a generation to be great" and that "you can be that great generation".

"An imbalance between rich and poor is the oldest and most fatal ailment of all republics."[7]

Plutarch (between 46 A.D. and 120 A.D.) teaches us that poverty already was the oldest and most fatal shortcoming of societies 2.000 years ago. Hence Nelson Mandela probably rightfully states that the generation who fixes this is a great one…

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[1] https://thenewinquiry.com/milton-friedmans-pencil/
[2] https://en.wikiquote.org/wiki/Carl_Sandburg
[3] https://en.wikiquote.org/wiki/George_Gissing
[4] https://idioms.thefreedictionary.com/one+today+is+worth+two+tomorrows
[5] https://www.youtube.com/watch?v=UxQfDLB4dfs
[6] https://www.one.org/us/blog/10-powerful-quotes-from-mandelas-make-poverty-history-speech/
[7] https://inequality.stanford.edu/publications/quote/plutarch

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Abstract

PART 1 TIME TO VALUE

Since the existence of Labour Theory of Value (LTV), developed by the classical economists Karl Marx, Adam Smith and David Ricardo over two centuries ago, it has been argued that labour (useful human time) is the only source of value. At the time economists could not use modern financial accounting, corporate finance and asset pricing techniques to build a mathematical framework on this premise. Today we can, which is exactly what we do in part 1 of this book. Based on the premise of LTV we derive a conceptual view on macroeconomics that is both simpler and more congruent with other parts of economics than the neoclassical school which has been dominating the economical debate since decades.

Neoclassical macroeconomics is based on the premise that there are several production factors, which always include labour and capital. The fact that “capital” as a production factor refers to “capital goods” is often disregarded; we need the capital goods for production, capital itself is just a legal claim on profits. In line with LTV, we argue that labour is the only factor of production because the costs of creating capital goods is exclusively determined by labour. The natural resources they were made off (if any) were already freely available for humanity at the time of production. Throughout the value chain there was no effort (“costs”) involved in creating the capital goods other than human time and humans never “paid” nature for anything. The same holds for consumption goods and (obviously) services. The value of natural resources then is the value they carry in a final useful state (either a consumption good or a capital good) minus the investment of human time that is required to get there.

Consumption of value occurs in two ways. Firstly, people consume goods (consumption of labour from the past) and services (instant consumption of labour). Secondly, people consume the opportunity costs of labour whenever they are not working (leisure time). Expressing both the costs and value of aggregated production and consumption in a closed economy in terms of human time instead of a monetary value forms the basis we use to derive a conceptual macroeconomic framework. In analogy with “the time value of money” in corporate finance we introduce the “time value of time” that states that present time is more valuable to humans than future time, which we use to discount future human time. Furthermore, we use the net present value method and the law of diminishing marginal returns to denote the value of a closed economy in terms of human time. This enables macroeconomic analysis of the real economy without considering the complex dynamics of our financial system and shows two important things:

1. There is an optimal ratio between (1) labour that is invested in creation and maintenance of capital goods at the one hand and (2) production of consumption goods and providing services at the other hand, that maximises the aggregated value of a closed economy like our globe.

2. What we consider to be the risk-free rate in asset pricing in fact reflects the depreciation rates of the remaining expected lifetimes of individuals and organisations in the real economy (interpretable as the annual chance to respectively die or default) who are considering buying assets. This “time value of time” is personal and hence differs for anyone that is valuing an asset. This individuality of the risk-free rate is currently not included in asset pricing theories.

Finally, we use financial accounting to develop the aggregated time statements (financial statements denoted in units useful human time at a reference year) of a closed economy. Time accounting and valuation show that in a closed economy wherein the people invest in increasing future labour productivity there is an aggregated free time flow at the disposal of the people in this economy which they can use to improve their “quality of life” by either (1) reinvesting in further increasing future consumption and/or (2) consuming more future leisure time (“time dividends”).

PART 2 MONEY TO SHARE

First, we relate valuation and asset pricing to the natural dynamics of value in the real economy as developed in part I of this book (Time to value). This shows that we can relatively easily include the individuality of the risk-free rate (time value of time) into our current models of corporate finance and asset pricing. The well-known net present value formula ("Value equals free cash flow divided by the cost of capital minus expected annual growth") remains unchanged, but the cost of capital now consists of (1) a premium for the inability of the investor to predict future growth (which reflects his individual exposure to uncertainty of expected future cash flows) and (2) a premium to account for the investors' time value of time (which reflects the risk aversion of the investor).

Secondly, we relate economics and financial accounting to time accounting. It reveals that free cash flows (dividends) in the financial system generally flow to different people (investors) than free time flows (dismissals and labour reduction) do in the real economy (employees). Therefore, the people that receive more free time often do not have the income required to spend their additional free time as leisure. This leaves them no choice but to find new jobs and keep working equal hours. Consequently, (nearly) all productivity increases have always been reinvested in consumption growth instead of leisure time.

Thirdly, we develop the consolidated financial statements (P&L, balance sheet and cashflow statement) of a fictive merger of all companies and financial institutions (referred to as the "private sector") of a closed economy with a certain Gross Domestic Product *(GDP=C+G+I)*, wherein all companies and financial institutions are privately owned and the government does not employ people but sources all its services from the private sector (referred to as a "truly capitalistic closed economy"). By consolidating the private sector we eliminate all business-to-business transactions, which leaves us with all business-to-consumer transactions (commonly referred to as *"C"*) and all business-to-government transactions (commonly referred to as *"G"*). This way, we denote the consolidated financial statements of the private sector of a truly capitalistic closed economy in terms of the various components *(C, G* and *I)* of GDP.

From these consolidated financial statements we derive among other things the "net public budget constraint". This is a numerical equation that expresses the development of the consolidated debt position of the public sector (all loans provided by the private sector to all governments and all households) in terms of the net interest rate *(r)*, nominal growth rate *(g)* and fractions of the various components of GDP. The net interest rate is the weighted average interest rate paid on public debt minus the fraction that is regained by the public sector by (1) imposing taxes on the financial sector and (2) by labour costs and dividend payments of the financial sector. We then solve the continuous-time equivalent of this numerical equation assuming a steady state. This shows that our current financial system is unstable if the net interest rate is higher than the nominal growth rate of the economy, which is probably not the case in real-life. However, by decomposing the public budget constraint it appears that our current system of money creation (fractional reserve banking) inevitably results in both financial instability and an ever-increasing inequality between households. A policy of inflation, which most central banks have, accelerates these dynamics.

Finally, we give some guidance on how we could adjust our monetary control and inheritance taxing policies to develop a sustainable financial system that is both stable over long periods of time and reduces inequality between households. Part 2 of this book comes with a spreadsheet model that supports the content of the book and can be used to see the long-term impact of tax regimes, nominal growth rates, interest rates and fractional reserve banking on financial stability.

1 Introduction

Most of my life I wondered how a beautiful system like capitalism and free markets that seem to perform reasonably well regarding the creation of wealth can deliver such poor results when it comes to the distribution of this wealth. For over a decade I thought that the main tenet of corporate finance (the risk-return relationship or the equity premium) also was the main flaw in the financial system. So, at the end of March 2018 I decided to spend a few months on researching and understanding how the underlying dynamics of asset pricing drive inequality. I wrote down two goals for myself:

1. Reject the Capital Asset Pricing Model (CAPM)[8]; and
2. Develop a theoretical framework that provides guidance to governments and central banks in facilitating both optimal value creation and fair distribution of wealth.

It wasn't until early December 2018 that I understood that I was wrong all the time. It appeared to me that I taught myself to value and account in terms of human time instead of money. This enabled me to separate the financial system from the real economy when observing economies. This made me understand that because free cash flows (i.e. dividend payments) generally end up with different people than free time flows do (i.e. dismissals), financial systems of capitalism-based economies are inevitably unstable without a redistribution system in place. With this book I tried to develop a basic mathematical framework that describes what I believe are the dynamics of value creation and distribution and relates this to the financial system we currently have.

Apart from various internet sources like Wikipedia and published scientific articles, I mainly studied the books listed below. I am grateful to the authors and everyone else who in any way contributed to these books.

- Thomas Piketty – Kapitaal in de 21ste eeuw, De Bezige Bij, achtste druk, 2017 ("**Piketty**");
- Kerry E. Back – Asset Pricing and Portfolio Choice Theory, Oxford University Press, second edition, 2015 ("**Back**");
- McKinsey & Company (Tim Koller, Marc Goedhart, David Wessels) - Valuation, Measuring and Managing the value of companies, John Wiley & Sons, Fourth Edition, 2005 ("**Koller a.o.**")
- Richard Brealey, Stewart Myers, Franklin Allen - Principles of Corporate Finance, McGraw – Hill Irwin, 8th edition, 2006 ("**Brealey Myers**")

I wrote this book for anyone who is familiar with macroeconomics, corporate finance and financial accounting and would like to be more aware about how our current economic thinking and acting in my opinion inevitably drives inequality and instability. You will notice that I am not a native speaker in English. Furthermore, this book is probably full of errors. This is all ok for me because it is a story with the purpose to improve the way we think about economics more than it is a document with a scientifically acceptable structure and content. I just made it up. So, if you agree to the content please pass it on. If you don't please tell me why so I can learn.

Either way, thanks for sharing!

Daan Koppen, December 2018

[8] The Capital Asset Pricing Model has been the most widely used and generally accepted model for asset pricing at capital markets since its introduction in the 1960s (Brealey, Myers, Back)

2 Summary

2.1 Time to value

2.1.1 Money has no value. Only time has

Finance does two things. It provides a marketplace that allows people and organisations to trade both in real goods and services and in their exposure to real risks and opportunities. We call this distribution. Secondly it is a ledger that accounts for all transactions between people and organisations within an economy. We call this registration. Finance is humanity's tool to enable efficient cooperation in maintaining and growing their economies. Finance does not create value itself; it is a zero-sum game.

The main priorities of people are to survive and to grow (make things better for themselves and others). In order to survive people spend part of their time (labour) by providing services and by transforming natural resources like matter and energy into useful consumption goods. The natural resources they use do not bear any universal value, they are just there. People also aim to grow by investing another part of their time (also labour) into creating capital goods, such that they can either (1) maintain an equal level of production in the future with less labour or (2) enjoy more future consumption by working equal hours. Capital goods include knowledge, human skills and expertise, social networks and alike. The efficiency gain obtained by investing in capital goods is the only driver of real economic growth per capita and is commonly referred to as labour productivity increase.

In this line of thinking we can denote the costs of all raw materials and (semi-)finished goods (which in fact are all services) in terms of human time spent (historic costs accounting method). The value of these goods can also be denoted in terms of human time; it equals the net present value of the expected future productivity gains (labour savings) the goods will bring minus the net present value of the costs (amount of labour) that remain required to get the goods into a final useful state. This way we can separate finance from the real economy which has the benefit of analytical convenience and allows us to disregard complex financial dynamics when observing economies.

2.1.2 The Zen of aggregated value

The value of a well-functioning[9] economy *V(s)* at the end of year *0* that invests a fraction *s* of its aggregated production Y_i into capital goods at any given year *i* in the future is described by formula (4.24).

$$V(s) = \frac{(1-s)Y_0}{(\delta_0+\delta_T-g(s))}, which\ equals\ V(s) = \frac{(1-s)Y_0}{(\delta_T-g_r(s))}, with\ g(s) = \delta_0 + g_r(s)\ if\ s > s_0 \qquad (4.24)$$

Y_0 is the aggregated production of the economy in year *0* (present time) denoted in units labour of reference year *0*, δ_0 is the weighted average depreciation rate of all capital goods in the economy, δ_T is the depreciation rate of human time that is the average of the (negative) drifts of the expected remaining lifetimes of all individuals within the economy which are assumed to evolve as Brownian motions. The depreciation rate of human time reflects the fact that present time is more valuable to humans than future time[10] (our time today is given, tomorrow you might not be there). Finally, *g(s)* is the annual growth of the economy which is a function of the investment rate *s.* Note that part of the investment rate s_0 is required as maintenance investments to avoid decay of the capital goods such that $g(s_0) = \delta_0$. So real growth $g_r(s)$ which equals *g(s)* minus δ_0 occurs if *s* is larger than s_0. For formula (4.24) to be valid δ_T must be larger than the real growth $g_r(s)$ and the people must use all future time released by productivity increases as labour (and not leisure)[11].

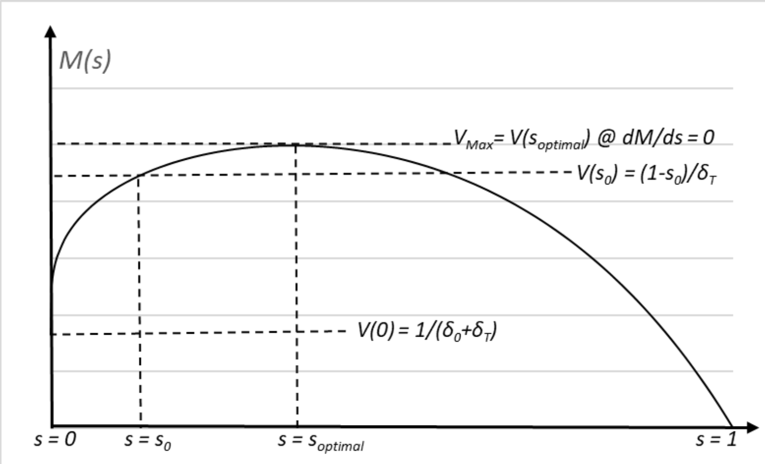
[9] Well-functioning means that (1) the people in the economy do the best they can and work together efficiently, (2) jobs are allocated to people who are best fit to perform them and (3) the people set rational priorities regarding investing in capital goods.
[10] δ_T is the equivalent of the risk-free rate in corporate finance.
[11] In asset pricing this is called a self-financing process, which means that all dividends are reinvested

2.1.3 Diminishing marginal returns and the optimal investment rate

If we apply the law of diminishing marginal returns it seems plausible that *g(s)* can be written as (4.29), which we can use to rewrite *g(s)* in formula (4.24).

$$g(s) = g_{max}s^{\gamma}, 0 \leq s \leq 1 \text{ and } 0 < \gamma < 1 \qquad (4.29)$$

Now we can divide $V_S(s)$ by Y_0 and rewrite equation (4.24) to present the value of an economy as a multiple *(M(s))* of the aggregated production at present *(Y_0)*, such that $M(s) = V_S(s)/Y_0$. Such a function is plotted below.



The figure shows that the value of an economy has a maximum at the optimal investment rate. This value can be derived if we differentiate *M(s)* to *s* and solve this for *dM/ds=0,* like formula (4.31).

$$\frac{dM(s)}{ds} = \frac{d}{ds}\left(\frac{1-s}{\delta_0+\delta_T - g_{max}s^{\gamma}}\right) = 0 \qquad (4.31)$$

2.1.4 Aggregated time accounting and quality of life

Please find below the aggregated time statements of a closed well-functioning economy for years *0* to years *i*. Assets are capitalised at “historic costs”.

Aggregated profit & loss account				
Line item	**Operator**	**Year 0**	**Year 1**	**Year i**
Revenues	+	Y_0	$Y_0(1+g)$	$Y_0(1+g)^i$
Opex	-/-	$(1-s)Y_0$	$(1-s)Y_0(1+g)$	$(1-s)Y_0(1+g)^i$
EBITDA	=	sY_0	$sY_0(1+g)$	$sY_0(1+g)^i$
Depreciation	-/-	δ_0Y_0	$\delta_0Y_0(1+g)$	$\delta_0Y_0(1+g)^i$
NOPLAT	=	$(s-\delta_0)Y_0$	$(s-\delta_0)Y_0(1+g)$	$(s-\delta_0)Y_0(1+g)^i$

Table 4.4 Aggregated profit & loss account denoted in units of useful human time in the present year (aggregated production Y_0)

Aggregated time flow statement				
Line item	**Operator**	**Year 0**	**Year 1**	**Year i**
EBITDA	+	sY_0	$sY_0(1+g)$	$sY_0(1+g)^i$
WC Adjustments	-/-	0	0	0
Capex	-/-	φsY_0	$\varphi sY_0(1+g)$	$\varphi sY_0(1+g)^i$
Free Time Flow (FTF)	=	$(1-\varphi)sY_0$	$(1-\varphi)sY_0(1+g)$	$(1-\varphi)sY_0(1+g)^i$

Table 4.5 Aggregated time flow statement denoted in units of useful human time in the present year (aggregated production Y_0)

Aggregated Balance sheet				
Line item	**Operator**	**Year 0**	**Year 1**	**Year i**
Assets				
Capital goods at the beginning of period	+	*0*	$(\varphi s\text{-}\delta_0)Y_0$	$(\varphi s\text{-}\delta_0)Y_0 + \ldots + (\varphi s\text{-}\delta_0)Y_0(1+g)^{i-1}$
Investments (capex)	+	φsY_0	$\varphi sY_0(1+g)$	$\varphi sY_0(1+g)^i$
Depreciation	-/-	$\delta_0 Y_0$	$\delta_0 Y_0(1+g)$	$\delta_0 Y_0(1+g)^i$
Capital goods at the end of period	=	$(\varphi s\text{-}\delta_0)Y_0$	$(\varphi s\text{-}\delta_0)Y_0 + (\varphi s\text{-}\delta_0)Y_0(1+g)$	$(\varphi s\text{-}\delta_0)Y_0 + (\varphi s\text{-}\delta_0)Y_0(1+g) + \ldots + (\varphi s\text{-}\delta_0)Y_0(1+g)^i$
Liabilities				
Equity at the beginning of period	+	*0*	$(\varphi s\text{-}\delta_0)Y_0$	$(\varphi s\text{-}\delta_0)Y_0 + \ldots + (\varphi s\text{-}\delta_0)Y_0(1+g)^{i-1}$
NOPLAT	+	$(s\text{-}\delta_0)Y_0$	$(s\text{-}\delta_0)Y_0(1+g)$	$(s\text{-}\delta_0)Y_0(1+g)^i$
Dividend (FTF)	-/-	$(1\text{-}\varphi)sY_0$	$(1\text{-}\varphi)sY_0(1+g)$	$(1\text{-}\varphi)sY_0(1+g)^i$
Equity at the end of period	=	$(\varphi s\text{-}\delta_0)Y_0$	$(\varphi s\text{-}\delta_0)Y_0 + (\varphi s\text{-}\delta_0)Y_0(1+g)$	$(\varphi s\text{-}\delta_0)Y_0 + (\varphi s\text{-}\delta_0)Y_0(1+g) + \ldots + (\varphi s\text{-}\delta_0)Y_0(1+g)^i$

Table 4.6 Aggregated balance sheet denoted in units of useful human time in the present year (aggregated production Y_0*)*

In the statements above φ is the investment rate, which is the fraction (percentage) of EBITDA (sY_i) that is reinvested in creating future value.

We can now define the annual increase (or decrease) in quality of life in year *i* (accounted for based on historic costs) as the amount of labour investment in capital goods minus the depreciation of capital goods $(s\text{-}\delta_0)Y_i$ during the period *i*.

This equals $NOPLAT_i = (s\text{-}\delta_0)Y_0(1+g)^i$ which is the equivalent of the net profit of an unlevered company in financial accounting. The time dividend or free time flow $DIV_i = FTF_i = (1\text{-}\varphi)sY_0(1+g)^i$ is the share of the increase in quality of life that was spent on increased future leisure time. The remaining part $(\varphi s\text{-}\delta_0)Y_0(1+g)^i$ is reinvested in increased future consumption which is accounted for by adding this to the equity reserves of the economy valued at historic costs.

Formula (4.24) assumes that all productivity increases are reinvested in the economy (a self-financing process which implies φ equals 1). We did not take leisure time into account. Obviously, once people enjoy more leisure time (time dividend) this impacts the growth of future consumption. Therefore, we should account for this by including the fraction φ of available labour sY_i that is used for creation of capital goods. If we adjust formula (4.24) we get formulas (4.32) and (4.33):

$$V_s = \frac{(1-\varphi s)Y_0}{(\delta_0+\delta_T-g(\varphi s))} \Rightarrow V_s = \frac{(1-\varphi s)Y_0}{(\delta_T-g_r(\varphi s))}, if\ s > s_0 \qquad (4.32)$$

$$M = \frac{V_s}{Y_0} = \frac{(1-\varphi s)}{(\delta_0+\delta_T-g(\varphi s))} = \frac{(1-\varphi s)}{(\delta_0+\delta_T-g_{max}\varphi s^{\gamma})}, with\ 0 < s < 1, 0 < \gamma < 1\ and\ 0 < \varphi < 1 \qquad (4.33)$$

If φ is smaller than 1, formulas (4.32) and (4.33) describe the value of the quality of life by means of consumption (at fair value instead of historic costs) and disregard the value of time dividends. The value of time dividends is simply obtained by subtracting formula (4.32) from formula (4.24), which equals the summation of (4.32) when φ is replaced by $(1\text{-}\varphi)$, which describes the summation of an indefinite series of free time flows valued at their future level of quality of life compared to present time discounted by the depreciation rate of human time.

The annual aggregated production is commonly denoted as $Y_i = A_i L_i$ wherein Labour (L_i) is a function of population growth and the time dividend rate $(1\text{-}\varphi)$ to account for adjustments in the annual working hours multiplied and A_i is a production efficiency function that is driven by the investment rate *s* and the labour productivity growth. By separating labour and productivity we can monitor efficiency gains and population growth separately.

Summarising it appears that people in a well-functioning economy that invest part of their labour supply in the creation and maintenance of capital goods increase labour productivity which they can use to improve their future quality of life. They can freely decide whether they spend the annual increase in quality of life on (1) more future leisure time or (2) more future consumption.

2.2 Money to share

2.2.1 How time valuation relates to asset pricing

The risk-free rate is the depreciation of life expectancy with a premium for volatility

Transactions of capital goods in the real economy happen because different people have different views on the value of a capital good (or a set of expected future free cash flows). The individual value perception of a capital good is driven by (1) a plan (i.e. the expected future growth of the existing cashflows and the required investment rate), (2) the predictability of these cashflows and (3) the time perspective of the individual (time value of time). The value perception of an individual can be modelled with existing corporate finance techniques (like the net present value method), however we might want to reconsider the meaning of both the risk premium (equity premium) and the risk-free rate, that currently may not align too well with the dynamics in the real economy.

As an individual looks further into the future, she will less likely be alive and therefore she will value present time over future time based on her chances on survival. For example, imagine a person that has a 20% chance on dying every year. She will value next year at 80% compared to the value of this year, the year thereafter at 64% etcetera. Any given year *i* years into the future will be worth $(1-r)^i$ for her, equal to the chance she still lives that year. In this example *r* represents the chance on dying every year.

Young people that live a healthy and safe life have a longer expected remaining lifetime than old people or young people that live an adventurous life or live in a dangerous environment. We can possibly model the expected remaining lifetime $(T(t))$ as Brownian motion, with the depreciation rate of the expected remaining lifetime (δ_T) as (negative) drift. Both the drift and the volatility (σ_T^2) of the expected remaining lifetime are driven by the behaviour and environment of the individual and increase as she gets older. These dynamics do not fundamentally differ for companies that aim for acquisitive growth, except that the drift now reflects the (annual) chance on default and both the drift and the volatility are driven by (1) the sector dynamics and (2) the behaviour (like financial leverage) of the company. Based on this interpretation the risk-free rate in fact represents the “time value of time” of an investor instead of the return on an investment opportunity in a riskless asset.

The expected growth reflects the knowledge of the buyer with a discount for unpredictability

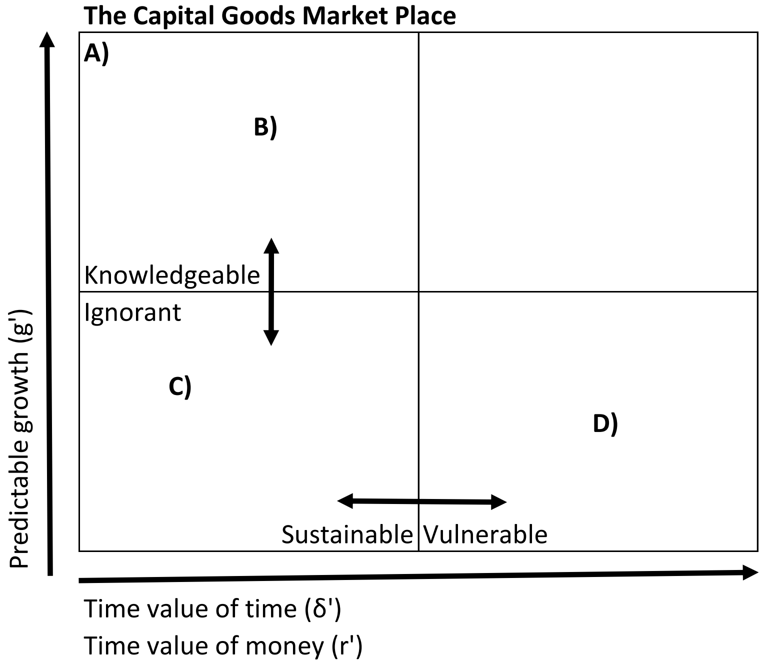
With this individual time value of time in mind the company (or individual or investor) observes a capital good or a free cash flow (both referred to as “asset”). Based on its own knowledge and specific sector dynamics the company projects a growth and required investment rate curve on the present free cashflow to estimate the expected future free cashflows. In analogy with the interest rate curve this also might be well modelled with Brownian motion, wherein the drift represents the expected future growth and the volatility reflects the level of unpredictability.

Net present value and the financial capital markets

If we now simplify both curves by assuming both the risk-free rate and the future expected growth constant over time and by modelling volatility as a premium on the risk-free rate and a discount on the expected future growth than this yields a formula equal to the well-known net present value formula that we obtain when we summarise the expected future cashflows to infinity. The difference is that the risk premium *r* now represents the sum of a discount μ_g regarding unpredictability of future growth and a premium μ_r regarding the volatility of the “time value of time” that is applicable to the buyer’s situation. Secondly, the risk-free rate represents the depreciation on the expected remaining lifetime of the buyer and not the return on an investment opportunity in a riskless asset. In formula we can write the following.

$$Value = \frac{FCF_{t=1}}{r-g} = \frac{FCF_{t=1}}{(r_f+\mu_r)-(g-\mu_g)} = \frac{FCF_{t=1}}{(r_f+\mu_r+\mu_g)-g} \quad (5.3)$$

Perhaps this interpretation can help explaining the "equity premium puzzle" and the "risk-free rate puzzle". The figure below shows how different organisations have different perspectives on the value of an asset. Investors in the left-upper corner attribute a high value to a specific asset and investors in the right-bottom corner attribute a low value to this asset. It shows that pension funds (C) for example should be more risk-neutral than highly-levered companies and trade buyers (B) have an advantage regarding growth and predictability.



Comments

g'= g - µg
r' = rf + µr (money)
δ' = δT + µT (time)

Value ≈ FCF/(r'- g')

A) Natural owner
B) Strategic buyer
C) Institutional investor
D) Seller

2.2.2 How aggregated time accounting relates to economics

Microeconomics

Every transaction within any economy is an exchange between time and money. This means that economic transactions yield to time flows in the real economy that move in the exact opposite direction like cashflows do in the financial system. Hence companies have both cashflows and time flows. Companies create value for humans by continuously increasing labour efficiency. The free time flows and free cashflows this process generates are partly reinvested in the company for ongoing economic growth. The remaining money (excess cash) and time (redundant labour) is distributed back to respectively the shareholders (dividend payments) and the employees (dismissals). However, because employees and investors generally are different people, the employees that receive time dividend (dismissals) do not have the monetary income (dividend payments) to spend this time as leisure. Therefore, most employees have no option but to work equal hours in the future[12] and virtually all productivity increases in capitalism-based economies always have been reinvested out of necessity into consumption growth instead of spending more leisure time.

[12] See also David Graeber's book Bullshit Jobs (Simon & Schuster, 2018), that argues that 25% of the jobs in western economies are considered useless by the people who occupy these jobs

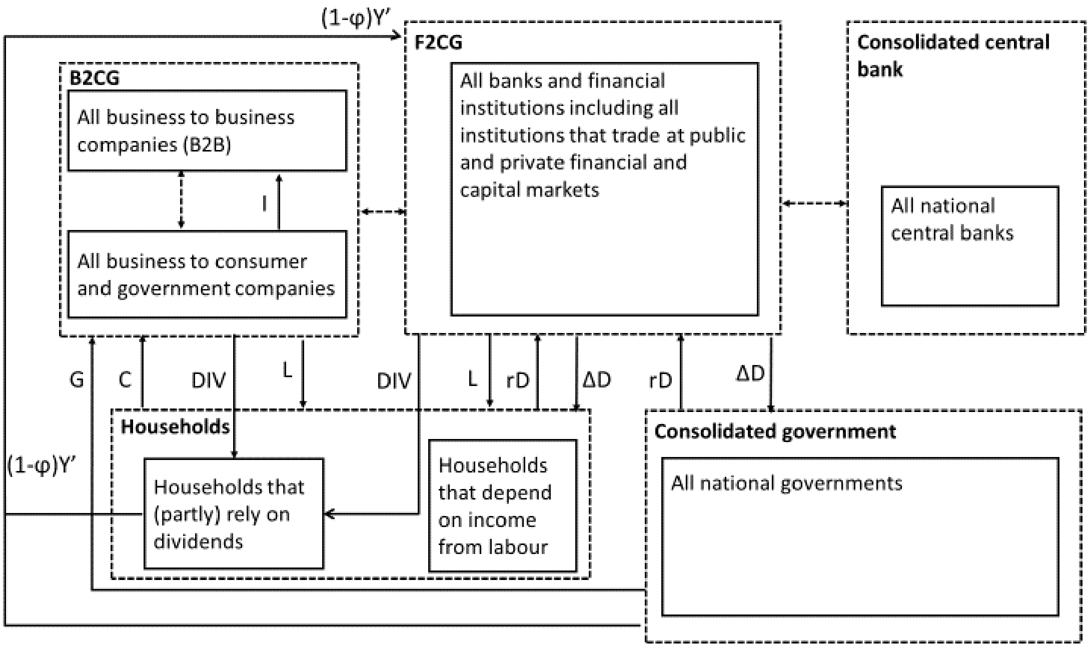
Macroeconomics

The equivalent of aggregated production in time accounting and valuation *(Y)* is the Gross Domestic Product in macroeconomics and the financial system. We should bear in mind though that there are some important differences which make them develop differently over time:

- In time accounting all human efforts should be included (like looking for a new job, housekeeping, raising children) whereas GDP only accounts for economic activity that is registered. For example, in time accounting unemployment can only be voluntary.
- In time valuation capital goods include human experience and skills, social networks, human knowledge and alike whereas in the financial system capital goods only include labour that was activated on companies' balance sheets.

2.2.3 How macroeconomics relates to financial accounting

Imagine a closed economy with a Gross Domestic Product *(Y)* which equals *C+G+I* wherein all labour is either employed by privately owned companies (jointly referred to as B2CG) or privately-owned financial institutions (jointly referred to as F2CG). In this economy governments do not employ people, but they source all their services from the private sector instead. Now assume a large global merger between all privately-owned companies (B2CG) and all banks and all other financial institutions in the economy (F2CG), which we will refer to as "the private sector". This truly capitalistic closed economy is visualised below.



If we would obtain the consolidated financial statements of the private sector we would eliminate all business-to-business transactions and positions of the closed economy, because they would all qualify as intercompany transactions and positions. This leaves us with all business-to-consumer *(C)* and business-to-government *(G)* transactions in the economy. Therefore, these financial statements represent all transactions and positions between the private sector against all households and governments (jointly referred to as the "public sector"). These financial statements are drafted below (allowing a few non-critical simplifications).

CONSOLIDATED PROFIT AND LOSS ACCOUNT OF THE PRIVATE SECTOR (B2CG AND F2CG)		
Financial accounting	**Macroeconomics**	**Operator**
Consolidated revenues	$Y_i(1-s) + r_iD_{i-1}$	=
Operational expenditures (opex)	$(1-\alpha-s)Y_i + (1-\alpha)r_iD_{i-1}$	-/-
EBITDA *EBITDA as % of revenues*	$\alpha Y_i + \alpha r_iD_{i-1}$ α	=
Depreciation *(Dep)* *Depreciation as % B2CG sales*	Depreciation (δY_i) δ	-/-
EBIT	$Y_i(\alpha-\delta)+\alpha(r_iD_{i-1})$	=
Interest costs (eliminated)	*0*	-/-
Corporate income tax (CIT) *CIT as % of revenues*[13]	$\tau_{CIT} Y_i + \tau_{CIT} r_iD_{i-1}$ τ_{CIT}	-/-
NOPLAT	$Y_i(\alpha-\delta-\tau_{CIT}) + r_iD_{i-1}(\alpha-\tau_{CIT})$	=

Table 5.3 Profit and loss account of the merger of B2CG and F2CG expressed in both macroeconomics and corporate finance metrics

CONSOLIDATED CASHFLOW STATEMENT OF THE PRIVATE SECTOR (B2CG AND F2CG)		
Financial accounting	**Macroeconomics**	**Operator**
EBITDA *EBITDA as % of revenues*	$\alpha Y_i+\alpha r_iD_{i-1}$ α	=
Corporate income tax (CIT) *CIT as % of revenues*	$\tau_{CIT} Y_i + \tau_{CIT} r_iD_{i-1}$ τ_{CIT}	-/-
Working capital mutations *(0)*	Changes in intermediate goods	-
Capital expenditures (capex) *Capex as % of B2CG sales*	Investments (sY_i) s	-
Free cash flow (FCF)	$Y_i(\alpha-s-\tau_{CIT})+ r_iD_{i-1}(\alpha-\tau_{CIT})$	=

Table 5.4 Consolidated cashflow statement of the merger of B2CG and F2CG expressed in both macroeconomics and corporate finance metrics

In the financial statements, s is a constant fraction of Y that is activated (at historic costs) on the balance sheet such that corporate investments I equals sY, δ is the depreciation rate of all capital goods owned by the private sector relative to Y, τ_{CIT} is the corporate income tax rate relative to revenues (either Y or rD), α is the gross margin (added value) of the private sector relative to Y, r is the weighted average public interest rate and D is the public debt level (sum of all household debt and all government debt).

2.2.4 Financial instability

From the financial statements of a truly capitalistic closed economy we can derive the **exact public budget constraint** (formula 5.25), which expresses the public budget deficit of year *i+1* (ΔD_{i+1}) relative to the Net Domestic Product ($Y' \equiv Y-\delta Y=C+G+sY-\delta Y\approx C+G$ if $s\approx\delta$).

$$\frac{\Delta D_{i+1}}{Y'_i} = g + \theta + r\left(\frac{\Delta D_i}{Y'_i} + \theta\,\frac{D_{i-1}}{Y'_i}\right) \quad (5.25)$$

The right side of the exact public budget constraint shows the various components that are funded by the public debt increase. The first term reflects nominal growth (g). The second term reflects the saving rate (Θ) of some households and part of the private sector which implies an equal fraction was borrowed (or withdrawn from their savings) by other households and governments to fund their consumption. The third term $(r\Delta D/Y')$ represents interest payments over last year's debt increase, due to delayed income of taxes and labour income.

[13] Due to the absence of depreciation in the financial sector τ_{CIT} should be a lower fraction of revenues in case of financial institutions. This is modelled properly in the supporting spreadsheet model but disregarded in the content of the book.

The fourth term *(rΘD/Y')* represents the "net interest costs" of the existing debt, which equals the interest costs minus the fraction *(1-Θ)(r(D/Y')* that was recaptured by the public sector (taxes, labour, dividends). Disregarding defaults, the exact public budget constraint converges to an asymptotic level of debt relative to *Y'* of *(g+Θ)/(g(1-r)-r Θ(1-g))* if *rΘ(1-g)* is smaller than *g(1-r)*, which is the case for realistic values of *g, Θ* and r. We can rearrange the exact public budget constraint into the **net public budget constraint** (formula 5.37) by introducing the **net interest rate** (formula 5.36).

$$r_{net}(\frac{D_i}{Y'_i}) = \left[r\frac{D_i}{Y'_i} + (\theta - 1)r\frac{D_{i-1}}{Y'_i}\right] \quad (5.36)$$

$$\frac{\Delta D_{i+1}}{Y'_i} = g + \theta + r_{net}\frac{D_i}{Y'_i} \quad (5.37)$$

The continuous time differential equation that is the equivalent of the net public budget constraint (assuming a constant net interest rate) has the following solution with boundary condition *x(t=0)=0 and x(t)=D(t)/Y'(t)*.

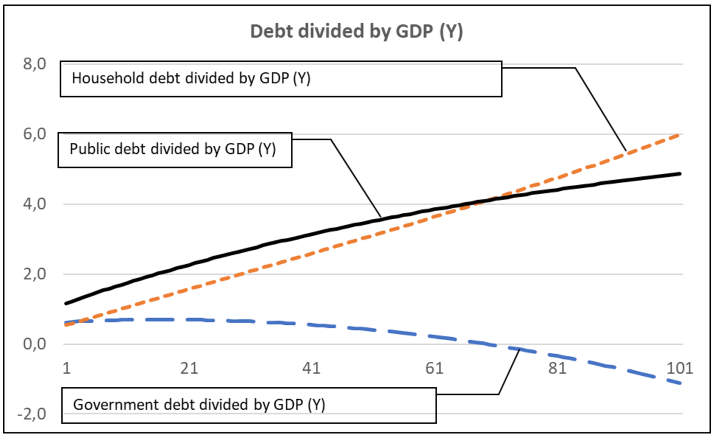
$$x(t) = \frac{D(t)}{Y'(t)} = \frac{g+\theta}{g-r_{net}}\left(1 - e^{(r_{net}-g)t}\right) \quad (5.38)$$

We can derive that if the net interest rate, the nominal growth rate, the profit margin, all tax rates, the dividend pay-out ratio and the savings rate are all constant over time, the net interest rate can be expressed as formula (5.40).

$$r_{net} = r(\alpha - \tau_{CIT})\left(1 - \frac{DIV}{FCF}\right) \quad (5.40)$$

Here τ_{CIT} is the corporate income tax rate and *DIV/FCF* is the pay-out ratio (dividend divided by free cash flow). The formula states that the net interest rate equals the public interest rate multiplied by the fraction that is withdrawn from the real economy and added to the excess cash of the aggregated financial markets. The net interest rate equals all rental income minus all payments to governments (taxes) and households (all labour costs of and all net dividends paid by the financial sector). It also confirms that under realistic conditions the net interest rate is smaller than economic growth such that the public debt level of a closed economy converges to an asymptotic value. Nonetheless, we should not conclude that the financial system of such an economy would then be fundamentally stable. This is because the aggregated public debt is a summation of all debt of individual households and governments, some of which are diverging positively (negative debt levels) and some of which are diverging negatively that jointly add up to a converging aggregated public debt.

To see this please find below a simple example using realistic variables wherein aggregated government debt and aggregated household debt diverge in opposite directions such that the aggregated public debt converges from present day debt levels for a period of a hundred years.



Although at aggregated level the financial system seems stable (converging to 5 times GDP) it is in fact unstable because it inevitably results in defaulting households. So, in order to understand financial stability we need to distinguish between various types of governments and households.

2.2.5 Financial inequality

The net government budget constraint

When decomposing the net public budget constraint, it can be shown that any governmental net budget constraint is expressed by formula (5.46).

$$\left[\frac{\Delta D^G_{i+1}}{Y_i}\right]_{\frac{\Delta D_{i+1}}{Y'_i}} = [\gamma g]_g + \left[\gamma - \tau_{CIT} + \tau_L(1-\alpha) + \tau_{DIV}\left(\frac{DIV}{FCF}\right)(\alpha - \tau_{CIT} - s)\right]_{(\theta)}$$

$$+\left[\left(\frac{r^G D^G_i}{Y_i}\right) - \left(\tau_{CIT}\left(\frac{r_i D_i}{Y_i}\right) + \tau_L(1-\alpha)\left(\frac{r_i D_i}{Y_i}\right) + \tau_{DIV}\left(\frac{DIV}{FCF}\right)(\alpha - \tau_{CIT})\left(\frac{r_i D_i}{Y_i}\right)\right)\right]_{r_{net} D_i / Y_i} \quad (5.46)$$

Here, the grey connotations refer to the equivalent terms of the net public budget constraint. This formula tells us that for some governments maintaining a sustainable budget is much easier than it is for other governments. The main components that make sustainable tax regimes easy for governments are (1) a net trading surplus and (2) a large and international financial sector. For governments of countries with a small financial sector and a trading deficit it is virtually impossible to maintain prudent budgeting except by inflating debt away (printing money to stimulate inflation).

The net labour-income dependent households' budget constraint

Imagine a group of all households in a closed economy that are fully dependent on income from labour and jointly spend a constant fraction c^L of the total GDP *(Y_i)* for any given year *i* such that their joint total consumption amounts to $C^L_i = c^L Y_i$ for any given year *i*. Let's also assume that their joint annual net income L^L_i is annually adjusted for inflation and real growth such that L^L_i is a constant fraction α^L of the GDP *(Y_i)*. This way the budget constraint of all the households that fully depend on income from labour in a closed economy is as follows:

$$C^L_{i+1} + rD^L_i = L^L_i + \Delta D^L_{i+1} \quad (5.47)$$

In words this means that the total consumption of the group of households that fully depend on labour income plus their joint interest payments must equal their income from the prior period plus the amount of new debt they need to borrow to fund the gap. The equivalent continuous time differential equation with *$x(0)=0$* wherein *$x(t)=D^L(t)/\alpha^L Y(t)$* is the amount of debt of this group of households relative to their joint net income has the following solution:

$$x(t) = \frac{D^L(t)}{\alpha^L Y(t)} = \frac{g}{g-r}\left(1 - e^{(r-g)t}\right) \quad (5.50)$$

Obviously, equation (5.50) disregards defaulting and holds true only for households that fully depend on income from labour and consistently fund nominal growth and interest obligations by borrowing money. Nonetheless it does reveal the main difference between households that fully depend on income and the rest of the public sector (households that own equity and governments). They are unexposed to financial income. Therefore, they have no feedback loop at all from their interest payments. Consequently, their net interest rate *(r_{net})* equals the interest rate they pay for their loans. So in order to maintain a sustainable financial position, the interest rate they pay must be lower than nominal growth. And we all know that this is not the case. Even interest rates on mortgage-backed securities generally exceed nominal growth, let alone all other forms of consumer credit. So even if wages of indebted households without exposure to further career opportunities would grow in line with nominal GDP growth, their growth of consumption must be lower to avoid defaulting. Equation (5.50) captures the essence of debt-financed growth; it is not sustainable unless debt is for free without repayment obligations, which would make it a gift rather than a loan.

If wage increases lag nominal economic growth (which has been the case for most western economies in the past decades) the income-dependent households budget constraint is given by equation (5.54), wherein g^L is the annual wage increase and $\alpha^L_0 Y_i$ is the total net income of these households in year *i.* The last terms on the right side between brackets express the ever-increasing additional funding requirement due to lagging wage increases.

$$\frac{\Delta D_{i+1}^L}{\alpha_0^L Y_i} = g + r\frac{D_i^L}{\alpha_0^L Y_i} + \left[1 - \left(\frac{1+g^L}{1+g}\right)^i\right] \tag{5.54}$$

Inflationary fractional reserve banking and inheritance are main drivers of instability

From the public budget constraint (5.25) we can see that there are two drivers in our current financial system that make the system fundamentally instable. Firstly, due to fractional reserve banking (money creation delegated to commercial banks by issuing loans) the creation of money to fund growth of public consumption is booked against new public debt *(gY'_i)*. This way, all real and inflationary growth of *NDP (Y')* creates an equal increase in public debt.

Secondly, since not all free cash flows are reused for consumption there is a drain of money *($\Theta Y'_i$)* out of the real economy into the financial markets. This amount is also booked against new public debt and hence further drives growth of public debt. Without central banking interference both drivers would inevitably result in deflation and negative nominal growth.

However, western central banks aim for nominal growth including inflation by stimulating borrowing (low interest rates) and injecting money into the financial markets (quantitative easing) instead of directly injecting money into the real economy where the money shortage occurs. Measures that in my view accelerate the increase of public debt and abundance of money in the financial markets. This can only end in negative interest rates and/or public defaults. The artificial growth of the financial sector driven by fractional reserve banking also yields to (1) a brain drain away from the real economy, which slows down real economic growth and (2) an ever-increasing amount of household savings and private sector excess cash, which inflates values of assets in the financial markets.

The third driver of instability in the financial system is low inheritance tax. Whereas debt is largely transferred publicly from generation to generation by government debt, equity and savings are largely inherited through bloodlines. Because return on capital is structurally higher than nominal growth *($r_{capital} > g$)* it enables rich families to live from return on capital and still transfer more wealth than they inherited to the next generation.

2.2.6 So, now what?

In order to develop a financial system that is both stable and ensures fair distribution we should reconsider the way we create money and the way we tax inheritance of capital. Central banks might want to consider annually withdrawing an amount of *$\Theta Y'$* minus a natural increase to compensate aggregated capital gain away from the financial markets by issuing riskless bonds to maintain a balanced amount of money in the financial markets relative to the aggregated value of assets. It could then estimate real growth based on labour volume and productivity increases and proportionally deposit an amount of *gY* on bank accounts of households and government as a gift to maintain price stability (preferably without inflation). Arguably, another part should be reinjected this way to cover for the drain of money from the real economy due to savings (*$\Theta Y'_i$)*. This way, the public sector can grow its level of consumption in line with economic growth without borrowing from the private sector, which yields to debt markets driven by natural dynamics (i.e. size and interest rates based on the time value of time and the chances on default). Secondly, annualised inheritance tax rates on capital should be larger than the spread between capital return and nominal growth, such that inherited capital gradually transfers to public ownership over generations. Preferably, inheritance tax is paid in kind, such that the public sector is increasingly exposed to return from capital and public wealth is protected against inflation.

These measures yield dynamics that will drive the financial sector into a sustainable situation with converging inequality, whilst maintaining the incentives for entrepreneurship that currently work so well in capitalism-based economies.

PART 1 TIME TO VALUE

3 Money has no value. Only time has.

3.1 A fictional history of finance to celebrate its importance

With the most recent financial crisis still fresh in our minds, it is easily forgotten how important finance[14] really is for the benefit of mankind. Therefore, allow me to fantasise a bit about how some of the most important financial milestones might have come about.

It helped me to realise the importance of the financial system in generating all the wealth we have today. Imagine a world without it. Secondly it supported my understanding that the sole purpose of finance is to serve humanity by enabling progress and creating wealth. No more, no less. And for sure, it is not the other way around. I refer to this as a “fictional” history of finance; not so much because it never happened, but because I did not spend any time fact-checking the story or the sequence of events. I just made it up.

So here is the history of finance.

Once upon a time an enlightened soul proposed the concept of money to his tribe members. This enabled mankind to trade and cooperate (divide tasks) efficiently because they now agreed on a central good (gold for example) which they called money that could be used to trade any other good or service and/or to keep track of favours owed to one another. How great is that? No longer did one need to pay in breads, just because he happens to be a baker. Or pay with apples if you grow them in your back yard. Basically, money enables specialization and cooperation within communities. Humans can now divide tasks and rely on each other.

Then came the concept of loans and debt. What a beautiful financial instrument that is. An older member of the community that wishes or needs to retire after making a good living now can support younger community members and (if things work out well) will get his money back later on along with interest payments as a compensation for the risk and trust. With the loan available these ambitious youngsters now can decide a profession and invest in required equipment and working capital. They can invest in a windmill if they want to be a miller, an oven should they prefer being a baker or buy land if they want to farm. In the meantime, elderly people can enjoy a well-deserved retirement living from interest and redemption payments.

Then a quantitative econometrist avant-la-lettre invented insurance policies. How beneficial? Entrepreneurs now can mitigate risks if they like. I can imagine, investing in a windmill is a lot easier if you know that you and your family will not be ruined the rest of your life if the mill accidentally burns down. Risks are now shared by a group of people.

If we can share risks, why not share in the benefits too? The concept of shareholder was introduced. Next to the risks, the benefits of a project or company could now also be shared between a lot of people (investors) instead of one (the entrepreneur). This sharing of risk and reward by raising capital from many people opened the gate for much larger ventures than any individual community member could ever have realised. This concept of shareholding was invented in 1602 when the Verenigde Oost-Indische Company was established (at least in The Netherlands we like to think so), which brought great wealth to The Netherlands in the 17th century. The VOC is generally considered as the first large international corporation (again, at least in The Netherlands we like to think so).

Since then we have been unable to introduce new financial concepts that really helped anyone in business. Nonetheless, we have been developing, shifting, repackaging, combining, hedging and categorising risks and rewards repeatedly ever since until recently when we all except a few exceptionally smart people were confused rather than convinced. Eventually in September 2008 the global financial system was at the verge of a total collapse…again. The rest is history.

[14] Throughout this short book I will use the term finance often. I am reluctant to provide definitions, because they are lengthy and boring. It will be increasingly clear during this chapter that with finance I mean all **activities** regarding distribution of value and registration of transactions within the economy. I do not mean specific sectors, organisations or people.

3.2 Finance is our system to register and distribute value

3.2.1 Debt and insurance

Now we have our fictive history of finance let's see if we can get a better understanding of what the various financial instruments we discussed really are.

Let's start with debt and insurance, they are easiest to understand.

Debt (a loan) is an agreement between to individuals (or organisations) regarding (1) the amount that is provided, (2) the interest rate, (3) the redemption scheme and possibly some other terms like security for the lender (collateral, mortgage, pledges, margin calls). So, debt is a **claim** that the lender has on the borrower. Please note that for every loan provided must hold that the value of this loan for the lender is equal to the liability of the borrower. It is just a transaction; no value is created. Therefore, if we would consolidate (add) all loans in the economy that are supplied at any given time it must be equal to the sum off all liabilities against these loans. The aggregated value is always 0.

Secondly, we have the concept of insurance which is another **claim** between two people or organisations within the economy, saying: "you pay me an amount every year and for that I will accept certain risks of your company or project whereas you get to keep the upside." The value of any insurance policy that is accounted for by the supplier should always equal the liability registered by the buyer of this policy. The value of the risk that was insured in the real world did not change by the transaction. Therefore, also the aggregated value of all insurance policies that is accounted for at any given time should equal the aggregated liability of the counterparties (buyers of insurance). Let's denote the aggregated recorded value of all loans and insurance products provided at any given time i by "$\boldsymbol{D_i}$". The aggregated value of all counter positions of these products will then always amounts to $-D_i$ such that the aggregated recorded value of all these positions is always 0. This must be true since against any position exists an equal counter position.

3.2.2 Money

The way I see it, money is just another a simple contract between the owners of money and the rest of the community that provides the owners of money in this community to right to receive goods or services from the community valued at the amount of the money they own. Since the contract only agrees on the value of goods and services, it is subject to mutual consent about all other terms and conditions. So, one monetary unit like a US Dollar[15] (USD) is a **claim** that the owner of the dollar has on the community to provide goods or services. So, the owner of the money provided a loan with no interest that is instantly due (repayable) in services or goods at the discretion of the owner, provided that the financial markets are liquid. The last comment is important, because the community will only respect its obligation if they believe the money represents value for them. Hence the importance of trust and a well-functioning legal system that respects the concept of ownership.

Like debt and insurance for every US Dollar owned there exist an equal liability in the economy. It is just that because we do not know yet how the money will be spent that we fail to account for these liabilities. We do know though that all the money in the economy must be spent within the economy After all, we cannot trade with aliens, animals, plants or earth itself. Therefore, the size of the aggregated liability is equal to all money owned by the people in the economy (let's call this "**M**"). When consolidating value, for example when we look at value at aggregated economic level we should therefore bear in mind that against all money with a total value of M, there is an equal liability. What it means is that money is just a tool to trade (distribute) and to record value. It is not a useful consumption good that has value itself like food.

[15] Since the United States dollar is the primal currency on the planet I will use this as a currency. Obviously, where it states USD it could as well be a EURO, Renminbi or any other currency.

3.2.3 Equity

Finally, we should look at what equity really is. With equity I mean ownership of (part of) any possible real asset in the economy like real estate, land, tangible capital goods like machines and factories or intangible capital goods like software, brand names and intellectual property. This could be indirect (like shares in a company) or direct, like an individual that owns an asset (collection of chain saws, a holiday house or financial expertise) that she commercially exploits (renting chain saws, renting the holiday house or selling financial advice). To keep things simple though we will assume that all real assets are indirectly owned by one or more shareholders, such that individuals owning an asset are considered companies with a single shareholder (sole proprietorships).

As widely acknowledged in corporate finance the value of (the shares in) a company is the net present value of the expected future cashflows that are available to the shareholders. Capital in the form of equity hence essentially is just another type of claim that people (in this case the shareholders) have on other people (in this case the present and future users of the company's assets) on a part of the value that is yet to be created in the future. This is a compensation for the owners of the shareholders for investing in the company's assets, which could be anything like houses, bridges, machines, equipment, software or human expertise. The company either bought this right from someone else or took the effort (paid for labour and goods) that was required to create the capital good. The only exception might be land and natural resources, which at some point in history just ware taken by or given to the initial owners. However, also land and natural resources generally require an effort before it pays off. This could be activities like drilling oil wells, digging mines or growing crops.

To summarise, equity is the ownership of capital goods that represent a claim on part of the future benefits the capital goods will bring in the future.

The value of all equity available ("$\boldsymbol{E_i}$") at any given time *i* is the net recorded value of all ownership of real assets within the economy. It represents the joint claim on future compensation that will be paid by the users (customers) of these capital goods to the owners. Again, please note that equity represents a claim on real capital goods (i.e. physical assets as opposed to financial instruments) with a value at which the equity is accounted for in the financial system. It is not necessarily a "fair value" representation of the value of the underlying real assets. However, it reflects real value in a way that there does not exist a counterparty within the economy with an equal liability against the owner of the equity. The value of the equity represents a piece of the pie. The pie then is the total value of the economy, which size is the main subject of chapter 4. So, the consolidated (or aggregated) value of all equity (E_i) available at any given time does have a certain value, unlike debt, insurance and alike.

3.2.4 Any other financial instrument

Obviously there exist more traditional useful financial instruments for the benefit of companies and individuals that were left undiscussed like interest rate swaps and currency swaps. And finally, there exist all kind of products that are in fact there but are not so much obviously helping individuals and organisations in the real economy. Think of put- and call-options, collateral debt obligations, credit default swaps and a lot of even more exotic financial products that few people ever heard of and virtually nobody except a few exceptionally knowledgeable people understand, jointly referred to as the "if-you-can't-convince-them-confuse-them products". To my best knowledge, they all essentially just reallocate the exposure to risk and reward between people and organisations. Like insurance policies, it is about distribution of exposure to real risks and opportunities without changing any of these risks and opportunities the real economy is exposed to. Therefore, for any buyer of such product there exists a seller whose position (i.e. exposure to real events in the economy) is impacted by the exact opposite of the position of the buyer. Hence the net impact of all transactions of all products that were undiscussed is always zero. Therefore, the aggregated value of all these products in the economy at any time is always zero.

Let's redefine "***D_i***" such that it now also includes all claims from all products that were left undiscussed and have an equal counter position. If there exist any financial product that does represent a claim on real assets that was left undiscussed, we consider it part of "***E_i***".

3.2.5 Citizenships and taxes are equity

In this book we do not spend too much time on taxes and nations. Instead we consider corporate income tax as a claim of all the nation's citizens on the profits of all the companies within that nation. If the closed economy coincides with the nation the citizens' claim is just a stake in the shareholding of all the companies, at least within the context of this book. So, equity *(E)* includes the citizens' claims on company profits by means of corporate income tax. For other taxes like labour income tax and consumer related taxes like VAT the situation is similar, but not the same.

Countries also have the right to exploit its natural resources, which ultimately is a claim of the citizens on value related to this. This value materialises when companies buy concessions or otherwise pay for exploiting the natural resources. Like corporate income taxes we consider claims on natural resources by means of citizenship to be part of the equity *(E)*. Let's refer to both citizenships and taxes as **public equity** and refer to governmental debt (due to funding budget deficits) as **public debt**.

3.3 Finance enables value creation, but is a zero-sum game

Now we took a deeper look at money and all monetary instruments available we can now conclude that finance is solely about registration and (re-)distribution of value. So, finance does two things. It provides a marketplace that allows people to trade both in real goods and in their exposure to real risks and real opportunities. We call this distribution. Secondly it accounts for all the balances between the people and organisations that trade within an economy. We call this registration. It is humanity's tool to enable efficient cooperation in creating real value. Finance does not create value itself, it is a zero-sum game.

The aggregated recorded value of all claims and liabilities from all financial instruments accounted for by the financial system (commonly denoted with "**K_i**" from the German word Das Kapital) can be written like formula (3.1).

$$K_i = E_i + D_i + (-D_i) + M_i = E_i + M_i \qquad (3.1)$$

Wherein at any given time *i*, E_i represents the aggregated recorded value of all financial instruments that represent a claim on real assets in the economy, D_i represents the aggregated recorded value of products like loans and insurance that have an equal counter position and hence this value is (or at least should be) exactly offset by the aggregated value of all these counter positions $-D_i$. M_i represents the aggregated recorded value of all the money on saving accounts (and in socks) within the economy. Although not accounted for, we know that against all money M_i denoted in formula (2.1) there exists an unrecorded liability of a similar size ($-M_i$) within the real economy that exactly offsets the value of M_i.

3.4 Only time has value

3.4.1 Let's look at people like biologists observe bees

Now we isolated finance and consider it a zero-sum game we can disregard finance[16] when observing an economy to examine value creation. Just look at humans like they were bees. Bees don't have a financial system in place and yet their "economy" is functioning reasonably well. Like humans, different bees perform different roles. They collect nectar and transform it into honey and they build honeycombs to store it. Taking

[16] With disregarding Finance, we assume that we remove the total administration (all records of the financial system) such that companies and organisations (including banks, central banks, governments) jointly are considered one group of people and real assets, but no longer hold any positions against each other or have any ownership claims. So, the people who worked within the financial industry like banks, central banks (and the government) are actually still there (but perhaps no longer very busy).

finance out of the equation allows us to take a more physical look at our economy, like we are biologists looking at humans. Do that and you will see a bunch of busy people on a planet. Now, what are they doing and why? Let's disregard the philosophical part of these questions and jump directly into Darwinian conclusions. Their main priority is probably to survive, both as individuals as well as a species. Secondly, they likely try to make things better. Again, both for themselves as well as for future generations. Please note that these priorities align well with the scope of most companies. Surviving would be the equivalent of envisaging a "going concern principle" or to aim for "continuity" which virtually every company on the planet does. Trying to make things better in the future aligns well with another common company goal to create "long-term sustainable growth". Therefore, a logical next step would be to see if we can use corporate finance techniques to derive some theory around valuation of an economy. This, no surprise, is the exact goal of the next chapter. Before we get there though, we need to discuss a few more physical aspects and link them to their equivalents in finance and economics.

3.4.2 Labour is the fundamental driver of value

Now we dismissed finance we can no longer use a currency. So, we need to find a new (physical) quantity instead to denote the value of goods and services and to describe the value of an economy.

In this regard please note that the people in the economy we are observing must be somewhere within the universal boundaries of space and time and hence are subject to the universal laws of physics as we know them. Just like anybody else. So, the first resource they have available is time. For any individual this holds until she dies (let's skip religion too). For the species this holds until they are extinct. Secondly, the universe is filled with matter which are all resources available to the people within the economy. Most likely though, humans will largely focus on earth's natural resources. Thirdly, they can use all available energy like sunlight. Important to note in this regard is that mass is just a form of energy. From a physical perspective mass and energy are the same thing and related by the most famous equation on the planet, which is Einstein's $E= mc^2$. The relevance of this for our purpose is that we can get energy out of matter. This is what we do when we create nuclear energy and (sort of) when using fossil fuels like oil and coal for energy consumption. So, from now on we consider energy and matter equal and call it "**natural resources"**. This leaves the people in the economy with just time and natural resources to maintain and grow their economy.

Another important law in physics is the principle of conservation of mass (or energy). Things (i.e. matter and energy) in nature do not suddenly appear or disappear. Things are either in our universe or they are not. Things can however change, for example by reshaping, by chemical reactions (molecular scale) or by nuclear reactions (sub-molecular scale). This is important regarding our purpose of valuing an economy. What this implies is that the value of all the natural resources available in the universe for the population within the economy is exclusively determined by (1) the effort it will take for this population to transform the natural resources into a form that is useful (let's call these "**goods**") and (2) the amount of use the goods have for the population. Although "low hanging fruit" resources require less effort than other resources to transform them in goods, natural resources do not represent any value by itself; they are just there. The potential value of the natural resources then depends on the value of the useful goods they could be transferred into minus the resources required to do so. As we will see further on this value can be denoted in terms of just human time (labour). The concept is like what we would call "added value" in corporate finance, which equals labour costs and profit margin. To see that natural resources do not bear any fundamental value think of a bucket of gold at an indefinite distance away. This is the equivalent of a bucket of gold that does not exist. In the same way an oil well on the moon has no value to humans on earth. For them it takes more effort (human time) to transfer the gold and the oil to earth (transform in a useful state) than the use it would bring and therefore these natural resources have no value for humans on planet earth.

Likewise, if we are running out of proven oil and gas reserves to meet our energy demand, we have a few options. We can look for new wells (like deep see drilling), we can aim to find ways to exploit different natural resources (like wind and solar energy) or we can use chemistry to transfer other natural resources into oil. Either way, it will require a time investment of the people within the economy. With agriculture it is no different. By growing crops like corn, rise and potatoes we transfer water, sunlight and minerals from fertile ground into food. We milk cows, we keep livestock and prepare and cook meat. We do not just grab a chicken from the woods and swallow it. It all takes human effort. As a final example consider the raw materials that jointly draw up your own house. The house my family and myself happily live in was built in 1933. It mostly consists of bricks, cement, wood, iron and glass all of which are created out of natural resources that have been there for ages and are abundantly available on this planet. It is the result of the efforts from a lot of workers including carpenters, bricklayers and factory employees nearly eighty years ago that transformed the natural resources into a useful good that represents value to humans.

3.4.3 All products are services

In economics goods can be either products or services. Now we know that the value of all goods is solely determined by the effort it took to transform matter in useful goods, we might as well no longer distinguish between products (which are tangible or physical) and services. We could stretch this a little further, by stating that products do not exist in an economy. The value that goods carry is not determined by their weight. On the contrary, mostly weight is a liability. The value goods carry was created by the labour that was required to transform the matter into useful goods. These were all services.

3.4.4 Consumption goods for continuation and capital goods for growth

All goods described above like food and energy have the purpose to survive, the people use them to continue their activities, to stay alive and healthy. In analogy with economics let's denote these goods as "**consumption goods**". So, consumption goods are goods related to the people's primal goal to survive.

Now let's discuss how people use goods for their second goal, which is "making things better".

As far as I am aware there are two ways in which people can make things better. Firstly, it could mean that people create something that allows them to realise a similar output of goods with less effort. In other words, they spend some time today such that they require less time in the future enjoying a similar level of consumption goods. This could be anything like building a bridge, a spade, a grain mill, a factory or a bicycle. The point is that people invest time today to save time in the future. It is an efficiency gain regarding useful human time. This is in my view exactly like what economists would refer to as **Labour productivity increase**. Let's call the goods people create in this regard "**capital goods**".

So now we have consumption goods to keep the economy running and capital goods to make the economy more efficient by increasing labour productivity. Obviously, when investing in capital goods the people can decide whether they wish to spend the released time in the future as leisure time or to create more goods. This brings me to the second aspect of making things better, which is quality of life, like comfort or pleasure[17]. People could also invest time into products that do not so much improve efficiency but makes people happier or more comfortable. This could be anything like pain killers, clothing or jewelleries. Unfortunately, I do not have a clue how to assign value to this and to include this into the equation.

So, please note that often it is a mixture of both. Obviously, people's efficiency is related to their well-being. For example, people need leisure, sports and good sleep at night to perform well at work. In the end, only efforts with an aim to improve the quality of people's life that does not have any economical use has an exclusive focus on improving the "pleasure" part of well-being. This could for example be elderly care. And even then, this only holds to the extent that adequate elderly care does not have a positive impact on the well-being and agendas of their relatives, knowing that their family is safe and treated well.

Ultimately, I believe that it is fair to disregard happiness and comfort and assume that all investment in capital goods have the sole purpose of increasing future labour efficiency (i.e. it is meant to save time in the future), although I cannot prove it. Alternatively, we could keep in the back of our minds that this assumption

[17] In the 2008 course of Advanced Valuation at the Amsterdam Institute of Finance that I attended the lecturer, which was a Fontainebleau Professor and partner of McKinsey, defined value as "happiness"

implicates that all increases in quality of life other than more leisure time are now included in our definition of consumption. We will get back to this at the end of chapter 4, where we will include leisure time into our definition of quality of life. For now, we will assume that economic value and growth just reflect efficiency gains and volume growth. In line with corporate finance let's refer to this "making things better" as "long-term sustainable growth", or just "growth". Having said this **we can now conclude that all labour invested in capital goods has the purpose to gain efficiency by increasing future labour productivity which people can use to spend more leisure time or to consume more in the future**. As a result, all investments in capital goods as well as the future (expected) benefits of these investments can now be expressed in productive human time (which is labour) instead of money value. And if we find a way to properly discount these expected future benefits denoted in human time we could use this for valuation purposes.

For example, the costs of a robot that automatically manufactures certain goods can be denoted by adding all the labour hours throughout the value chain that were required to produce the robot. Starting from the time it took to extract the natural resources from the ground, to processing them into semi-finished goods to processing it into components to assembling and testing the robot to packaging and transferring all the way to installing and commissioning the robot. In accounting this would be referred to as the "historic cost price method".

Similarly, the production line or robot is expected to save labour in the future, which represents value to humanity that can be calculated by discounting the expected annual amounts of labour annually saved by the robot. The net present value[18] of the total labour that is expected to be saved by the robot minus the total hours spent to create the robot (historic costs) is the fair value of the robot denoted in units useful human time (labour).

Likewise, we can now allocate a fair value to all raw materials and semi-finished goods along the value chain, by determining the net present value of the productivity increase the goods in its final form will bring to mankind minus the net present value of the amount of human time (labour) that remains required to transform the semi-finished goods or raw materials into finished goods.

3.4.5 Annual consumption equals annual production of consumption goods

If we disregard changes in stock[19], we can now see that the annual consumption C_i of a closed economy in any given year *i* denoted in productive time equals the annual aggregated production of consumption goods in that same year. The annual aggregated production of consumption goods apparently equals the aggregated need of the people within the economy to maintain their economy at its current level of wealth (or to "survive"). This, with the economy at the current level of development, apparently required the aggregated amount of productive time they needed to produce these consumption goods and to meet their needs. This must hold since (1) we assumed no changes in stock (i.e. all consumption goods that were produced were also consumed within that year) and (2) all time invested in capital goods did not yet add to the level of consumption goods. All remaining time available to the population that does not qualify as labour (let's denote this with "**leisure time**" of "**free time**") is lost forever. After all, we cannot store time like we can store money on a bank account.

3.5 What are the benefits of disregarding finance?

3.5.1 Analytical convenience

Disregarding finance when describing and valuing economies has several benefits.

The main point is that it allows us to consider valuation and distribution as two separate processes. This makes things simpler, because it avoids confusion and allows us to isolate and discuss one topic at the time. If we think of value creation we should think in terms of human time and disregard any financial metric or dynamics.

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[18] The net present value method is the most commonly used method in corporate finance to calculate value
[19] Instead of disregarding changes in stocks we could also consider stock to be part of capital goods, with a return of 100% in the year they are consumed and 0% return any other year.

When understanding distribution, we should focus on the financial system to understand how people register and share the value they jointly created. To see imperfections and imbalances we should relate the real economy to the financial system and see how they both evolve and compare over time.

Secondly, time cannot inflate or deflate. At least in Newtonian mechanics it can't so this assumption should work until we travel with the speed of light. Nor can central banks interfere and inject more time in the system by implying monetary instruments like quantitative easing or adjusting the interest rate. So, by using time as a measure of growth there is no difference between real growth and nominal growth. There is only real growth which is expressed by the labour productivity of any given year related to the labour productivity of the reference year.

Thirdly, by disregarding the distribution side of valuation we disregard (for the moment) the dynamics of free and efficient markets that automatically (i.e. without central or governmental interference) drive fair pricing where demand- and supply-curves meet. These dynamics are commonly applied by free-market-promotors[20] as if it was a "universal acid" – a concept so powerful that it eats through about every traditional concept as a kind of universal truth. However, what the concept really does in my opinion is killing space for thorough discussion and analysis.

3.5.2 Time is all we have and yet we consume it constantly

The whole exercise I went through to disregard finance when looking at people and economies took me great effort (and lots of time), but gratefully made me more aware of the value of human time.

Disregarding finance shows that time is the only thing in nature that I believe should have value to humans, at least when it comes to economics. It shows that everything we do, or do not, is about allocating our time. We can never walk away from this. The way we choose to spend our time exclusively determines the impact we have in life. We should constantly be aware of the value of the time we expect to have left; it is in fact all we have left in our lives.

Moreover, denoting value in terms of time reveals that time cannot be stored like money on a bank account. We can never take a break in our life, we can never call for a "time out". We keep spending time constantly since the day we were born. Even more so, if human time is spilled it is wasted forever. This fundamentally differs from money, that keeps circulating in the economy. It should make anyone aware to live mindful and spend time wisely (either her own or anyone else's time). The only way to save time is to invest the time we have today in efficiency improvements of tomorrow. This is what we do when we aim for growth. On a personal level this would be to aim for continuous development, aim to consider every experience as a lesson. Such that we either succeed or learn. But we never fail.

[20] This phenomenon is also referred to as "market-fundamentalism" (https://en.wikipedia.org/wiki/Market_fundamentalism)

4 Time to value

4.1 The zen of aggregated value is like the zen of corporate finance

In this chapter we will derive a formula that describes the value of an economy on aggregated level denoted in useful human time (labour) at its present level of productivity. We will see that it the results are very similar to "The Zen of Corporate Finance" or the "Key Value Driver Formula" (albeit denoted in time instead of currency). Both terms and formula (4.1) are taken from Koller e.a.[21]:

$$Value = \frac{Free\ Cash\ Flow_{t=1}}{WACC-g} = \frac{NOPLAT_{t=1}\left(1-\frac{g}{ROIC}\right)}{WACC-g} \qquad (4.1)$$

This formula is well known in corporate finance and has a lot of equivalent forms in finance literature. It describes the value of a company at present time *t=0*, subject to certain conditions. More than calculating the exact value of companies its purpose is to show what the fundamental drivers of value of a company are. The formula is based on the net present value of the future expected cashflows that are available to the shareholders of the company. $NOPLAT_{t=1}$ means *Net Operating Profit Less Adjusted Taxes* one year from present time at *t=1*. This is the net profit after taxes in case the company has no interest-bearing debt. The *g* represents the annual growth of $NOPLAT_{t=1}$ and is assumed constant until eternity. *ROIC* means *Return On Invested Capital*. It is defined as the annual growth of the profitability of the company *(g times NOPLAT)* divided by the net investments that are required to achieve this growth. The net investments can be expressed as a fraction (percentage) of the net profit (NOPLAT) such that formula 4.2 holds. This fraction is called the Investment Rate *(IR)*).

$$ROIC = \frac{g*NOPLAT}{IR*NOPLAT} = \frac{g}{IR} \qquad (4.2)$$

In fact, the term *ROIC* should be *RONIC*, which means Return On New Invested Capital[22]. However, it makes no difference if we assume constant return on investment over time and a constant investment rate *IR(t) = IR*.

4.2 The key value drivers of an economy are invested labour, growth and depreciation

4.2.1 An economy that wants to grow needs to invest labour to create capital goods

From chapter 3 we know that if people want to grow their economy, they should invest time in the creation of capital goods instead of spending it on leisure or creation of consumption goods. Now let's investigate this some more and see how this impacts the growing economy. Also, please bear in mind that we assume a closed economy and disregard year-on-year changes in stocks. This way, as we assumed earlier, the annual aggregated consumption equals the annual aggregated production of consumption goods. Finally, which was left open in chapter 3, we will assume that all efficiency gains are reinvested into the economy. This means that the population in this model chooses to continue to work similar hours instead of spending efficiency gains on leisure time[23].

Let's denote the aggregated production of such an economy in year *i* as Y_i, where the subscript *i* denotes the year. We will assume no population growth for now. And let's assume that the people in the economy want to improve (or continue to improve) from year *0* onwards when their aggregated annual production amounts to Y_0. To do so they choose to invest a fraction (*s*) of their aggregated available production hours (labour) every year to find ways to grow. As discussed before, investing time could be anything like building infrastructure, inventing stuff and automating production lines. Also, this could be immaterial investments like fundamental research or writing software. Most of all, this includes investing in human competencies like experience, knowledge and skills. Obviously human competencies are increasingly important as the economy develops. I even consider building social relationships to be investing since social networks allow people to work together more efficiently. Imagine a world in which no one knows each other. All together we will consider all labour allocated to create the goods above as investing *(sY_i)* and the results of the activities above like real estate,

[21] Koller a.o. pages 61-62

[22] Koller a.o. page 273

[23] In asset pricing this is called a self-financing process, which means that all dividends are reinvested

transportation equipment, electronics, infrastructure, internet, factories, software, knowledge, human competencies, social networks, etcetera are jointly referred to as **capital goods,** like our definition from chapter 3. Since we are not focusing on distribution yet, it is irrelevant who invests time (provides labour) and who is the owner of the capital goods. For now, it is enough to assume that the people in the economy have a properly functioning system in place that fairly distributes this value between all people within the economy such that it motivates them to jointly maximise the aggregated value of the economy. Like bees do.

4.2.2 Depreciation is inevitable but can be offset by replacement investments

Now that we made capital goods tangible it is also easy to see that inevitably all capital goods gradually lose their value. We will denote the gradual decay of capital goods that inevitably occurs if people do not invest in maintaining the level of productivity improvement **depreciation**. The rate of depreciation of a capital good is either driven by its technical lifespan or economical lifespan, whichever expires first. The technical lifespan is the expected time a capital good performs before it breaks down, is lost (retiring knowledge) or does not work anymore despite proper maintenance. The economical lifespan of a capital good is the expected time the capital good is useful until an innovation makes the capital good abundant because there exists a better alternative. A good example of a class of capital goods which value typically expires economically rather than technically are consumer electronics like computers, laptops, cassette-recorders, VHS video players, tablets and mobile phones, etcetera. The machines still work but given the alternatives available are of no use anymore and hence do not represent any value anymore. Also, the lifespan of production lines, production techniques, software and transportation equipment like cars typically expire economically.

An example of a capital good which value typically expires technically is infrastructure like a bridge or a road. At some point the structure just is at the end of its life and needs to be rebuilt (for example due to metal fatigue, fractured concrete or worn out tarmac. The capital good just needs replacement before it collapses despite appropriate maintenance.

Immaterial capital goods like people's knowledge, competencies and social networks are also subject to depreciation. People forget things they learned, knowledge and competencies become outdated when economies develop, and people retire and develop careers. These dynamics obviously require continuous training and education of the labour pool to maintain the competence level of an ever-changing population. These capital goods can also expire economically or technically. The knowledge to create outdated equipment (like tape recorders or gramophone records) has expired economically. If an employee retires (or makes a career switch) her knowledge and skills expire technically. Finally, immaterial capital goods like brand names gradually lose their value without branding.

From now on we will refer to all these efforts that people annually invest in the maintenance of the performance of existing material and immaterial capital goods and the replacement of technically expired material and immaterial capital goods as **replacement investments.**

Obviously, the pace of depreciation varies enormously between capital goods, but nothing lasts forever. We define the expected annual level of depreciation of the economy as the weighted average depreciation of all the material and immaterial capital goods installed in the economy that would occur in a certain year *i* if no replacement investments were done as a fraction $\boldsymbol{1/(1+\delta_0)}$ of the total productivity that is lost in the economy as a result. So, if *s* would be *0* in year *i* (which includes zero replacement investments) the aggregated production Y_i in year *i* would be $Y_{i-1}/(1+\delta_0)$. Perhaps it would be more intuitive to define depreciation as $\boldsymbol{\delta'_0}$ such that Y_i in year *i* would be $Y_{i-1}*(1-\delta'_0)$ but defining δ_0 provides easier mathematics and there is no fundamental difference. The actual difference is that we now take the aggregated production of year *i* as a basis of the depreciation rate instead of the year before *i-1*. For example if δ' is 10% of Y_0, then this is similar to $\delta=\delta'/(1-\delta') = 10\%/(1-10\%)=11\%$ of Y_1.

Furthermore, we assume δ_0 constant over time. As a rule of thumb, we can say that if the annual depreciation is δ_0, the indicative aggregated expected lifespan of all the capital goods installed in the economy roughly amounts to 1 divided by δ_0. For example, if δ_0 is 5% per year, the expected aggregated lifespan of capital goods is roughly 1 divided by 5% is 20 years. I am aware mathematicians will not agree to this, but as a ballpark figure I think it serves well to give us a tangible idea of the average lifespan of capital goods at a certain depreciation rate δ_0.

4.2.3 Capital goods enable growth

Now assume that the aggregated annual result of all investments sY_i in the creation (and maintenance) of capital goods is that the productivity annually growths by a fraction (percentage) g times Y_i. We will assume that g is constant over time if the investment rate s remains unchanged. This assumption basically states that (1) the amount of opportunities for further productivity improvement remains constant over time so there is no endgame wherein humanity obtains absolute knowledge and (2) humanity has an unchanged intellectual capacity to capture these opportunities (i.e. ignores evolution of the human brain).

In other words, $g(s)$ is just a function of s and reflects the ability of the people in the economy to grow their aggregated production Y_{i+1} in year $i+1$ by investing a fraction s of their aggregated productivity Y_i in year i to improve their productivity. In formula, we wright:

$$Y_{i+1} = \frac{1+g(s)Y_i}{(1+\delta_0)} \tag{4.3}$$

We can now see that an economy that wishes to maintain its current level of aggregated production Y_0 in the future needs to invest at least a fraction s_0 such that the expected growth $g(s_0)$ equals δ_0.

To see this, we need to replace $g(s_0)$ by δ_0 in formula (4.3).

$$Y_{i+1} = \frac{1+g(s_0)Y_i}{(1+\delta_0)} = \frac{(1+\delta_0)Y_i}{(1+\delta_0)} = Y_i \tag{4.4}$$

This reflects the fact that an economy must invest an amount of s_0Y_i as replacement investments into its capital goods to maintain its current level of aggregated production. An economy is expected to grow if its investment rate s is larger than s_0, so the growth g is larger than the growth required to keep up with the depreciation rate δ_0. Since Piketty estimates that all continents on the planet showed long term average growth since 1820[24] I think it is fair to say that investment rates of mature economies are above the replacement level such that economies are expected to grow annually. Let's call this growth the **average expected real growth rate** g_r, such that $g(s) = \delta_0 + g_r(s)$, with $g_r(s)>0$ if $s>s_0$.

Finally, I note that we defined growth and depreciation as *average* rates. It means that within a certain timeframe the annual real depreciation and growth can deviate, driven by economic cycles. Also, the economy could suffer from setbacks like wars, natural disasters or adverse climate change effects. Obviously, the annual growth during wars or disasters could be even negative and depreciation could increase up to the rate all material goods are destroyed. What average expected depreciation and growth assume is that the economy ultimately is resilient to setbacks and assumes that the economy will always recover to maturity (or steady state) level of an expected long-term average growth of g and depreciation of δ_0, in which the poor years (and good years) are included.

4.2.4 A human life is like a stock price moving downwards

Now there is one remaining important topic that we didn't yet touch upon. We need to account for the fundamental uncertainty regarding the future. What it means is that present time is more valuable to humans than future time is, simply because their time today is a given. Tomorrow we can all be dead. Whereas δ_0 represents the expected (or even guaranteed) decay of both tangible and intangible capital goods (which includes human expertise) we will now introduce the **depreciation rate of human time (δ_T)** to account for the fact of life that time today is worth more to people than time tomorrow. Think of a human life as a random process with an expected remaining lifetime of the individual (**T**) and a variance ("$\boldsymbol{\sigma_T^2}$") that are both constantly

[24] Piketty page 116 table 2.5

impacted by (1) the behaviour of the individual during her life and (2) the environment of the individual. On the one hand people can decide to avoid risks which decreases the depreciation rate of their remaining life expectancy and reduces variance, or people could be more adventurous that would have an opposite effect. Everybody within the economy at any given moment in time has an expected remaining lifetime and variance that relates the value of their future time to their present time. Both the expected lifetime and the variance of every individual continuously transform driven by her behaviour and environment as she moves through life. Remaining life expectancy and variance can both increase and decrease in the short term. In the longer term the expected remaining lifetime decreases and variance increases until she dies, when both variance and expected remaining life are 0. The only guaranty in life is death (and taxes, as they say). Now, consider an individual remaining life expectancy *(T)* as continuous-time Brownian motion (like a stock price trajectory) with a negative drift that is the depreciation rate of time of the individual ***($\delta_{T,I}$)*** and a standard deviation ***($\sigma_{T,I}$)***. This is expressed by formula (4.5).

$$\frac{dT}{T} = \delta_{T,I}dt + \sigma_{T,I}dB \qquad (4.5)$$

The depreciation rate of human time (δ_T) that we described above and require for valuation of an economy can be derived like the way the expected return of the market portfolio can be derived from the expected returns of all stocks available in the market with stock prices described by Brownian motions. I will stay away from this[25], because to me it is rather advanced statistics. However, I encourage anyone to look further into this concept and would welcome any feedback regarding human lives as Brownian motions. We will consider the depreciation rate of human time **(δ_T)** as the average of the individual depreciation rates of time ***($\delta_{T,I}$)*** of all people in the economy and consider it constant over time. Which is not necessarily true, for example in case of group behaviour or natural disasters.

To make this whole concept more tangible please consider an individual that starts smoking when she is still young. Every cigarette that she decides to smoke reduces her life expectancy and increases the variance as she is now more exposed to various deceases. It decreases the value of her future simply because it is probably shorter than it would be if she were a non-smoker. In the mean-time her boyfriend is on a dangerous mission in space, which increased his depreciation rate of time. The moment he landed safely and decides to never go into space again his depreciation rate of time falls back to normal and immediately increases his remaining life expectancy. A few years later unfortunately she suffers from lung cancer with a 20% change on dying every year from the end of year 0 onwards. This means that she has an 80% chance that she lives through year 1, which reduces the value of year 1 to the level of 80% compared to the value of year 0. Year 2 for her is only worth $(80\%)^2$=64% of year 0 and so on.

Now since her life expectancy is roughly 5 years[26] and considering the impact this has on the value of her present time related to her future, it is unlikely that she retains a long-term focus when planning her life or career. With her boyfriend safe on the ground again, she will likely aim to get the best out of the time they still have together and not start a 10-year study to become a brain surgeon, what she dreamed of when she was still young.

The depreciation rate of human time is related to the expected average remaining human lifetime, which equals half of an average human lifetime. Therefore, the depreciation rate of human time should be a few percent in mature economies (relating to an average human life of say 75 years that is half way). Although it probably oversimplifies things as people do take the interest of future generations into account (for example

[25] Brownian motion is explained by Back, chapter 12, pg 289 to describe stock price movements, but originates from Physics. The concept was first described by Robert Brown in 1827. In 1905 Albert Einstein derived Brownian formula’s (like formula 3.5) to describe the trajectory of molecules driven by diffusion. Brownian motion in corporate finance could be considered as a biased tossing-game or throwing weighted dice wherein the level of bias represents drift and determines the pace that biased outcomes occur more often than the other statistical outcome(s)

[26] It would be better to say that she has a *$(100\%-20\%)^5$=33%* chance on surviving the next 5 years

when considering climate change) and people do value well-being of their children often over their own, it still might be a ball-park figure to keep in mind.

The depreciation rate of human time is the equivalent of the real (adjusted for inflation) **risk-free rate (r_f)** in corporate finance**,** which is the discount rate used in net present value calculations to account for the time value of money. To see this, please note that in net present value calculations future expected cashflows are valued at present by using a discount rate (which is the WACC in formula (3.1)) that reflects (1) the amount of risk of the future cashflows and (2) the time value of money[27]. So, if we eliminate the portion that reflects risk in the discount rate we are left with the so-called risk-free rate, which is defined as the annual return on an investment with a zero chance on default. Therefore, the real risk-free rate only accounts for what the lenders regard as their loss of value for not having the opportunity of instantly accessing the funds they invest. Hence, assuming well-functioning capital markets, the risk-free rate essentially represents humanity's view on the time value of money. Therefore, provided that the risk-free rate at the financial markets properly reflects the time value of money, our depreciation rate of human time δ_T equals the real (adjusted for inflation) risk-free rate r_f. Please note that the depreciation rate of human time can never be negative, hence a risk-free rate below zero (such as has been the case for a several years in the financial systems of Europe and Japan like negative interest on German and Japanese bonds) can never be natural until the day we can travel back in time.

If we include the depreciation rate of human time δ_T into formula (4.3) we obtain formula (4.6) that expresses the value of the aggregated production Y_{i+1} of year $i+1$ in terms of the value of the aggregated production Y_i of year i from the perspective of humans at time i.

$$Y_{i+1} = \frac{1+g(s)Y_i}{(1+\delta_0)(1+\delta_T)} = \frac{(1+\delta_0+g_{r(s)})Y_i}{(1+\delta_0)(1+\delta_T)} \approx \frac{(1+\delta_0+g_{r(s)})Y_i}{(1+\delta_0+\delta_T)}, if\ \delta_0\delta_T \ll \delta_0\ and\ \delta_T \qquad (4.6)$$

This formula dictates that the value of the aggregated production Y_{i+1} at year i is equal to the value of the aggregated production one year before multiplied by a factor $(1+g)$ due to expected growth of the economy, divided by a factor $(1+\delta_0)$ due to the depreciation in the economy of capital goods that negatively impact productivity and divided by a factor $(1+\delta_T)$ because Y_{i+1} is one year ahead in the future and therefore it loses a fraction $1/(1+\delta_T)$ of its value compared to present time, which is given. Please note that the denominator $(1+\delta_0)(1+\delta_T)$ can be written as $(1+\delta_0+\delta_T+\delta_0\delta_T)$. This is almost equal to $(1+\delta_0+\delta_T)$ if $\delta_0\delta_T$ is much smaller than both δ_0 and δ_T. As δ_0 and δ_T are both probably less than 10% this is largely true. In a continuous-time model equation (4.6) is exact. In Appendix 7.2 we use a continuous-time model to obtain an equal result. Arguably continuous-time models better reflect reality, so we can regard formula (4.6) to be accurate.

All we need to do now to obtain the value of an economy is to add the present values of aggregated productions of all years in the future all the way up to eternity. We will do this in the next paragraph. Before we do though we will briefly summarise what we discussed.

There are only three drivers of value; investment of labour, growth and depreciation. To grow their economy, people need to invest labour (useful human time) in capital goods. Capital goods are expected to increase future human productivity per unit time, which we call growth g. Capital goods depreciate at rate δ_0 if no labour is invested to maintain its functionality. If the investment rate s is at the level of replacement investment rate s_0 the annual expected average growth is equal to the annual average expected depreciation rate δ_0 such that the average expected aggregated production of an economy remains unchanged at every given year in the future. If the investment rate s exceeds the replacement investment rate s_0 the economy is expected to grow with $g(s>s_0)=\delta_0+g_r(s)$, wherein $g_r(s)$ is the real growth.

Finally, we need to consider that present human time is more valuable than future human time, simply because our time now is a given. To do so we consider and individual human life as an expected remaining lifetime with a variance that both are continuously affected by the individual's behaviour and environment as her time passes by. This is like a Brownian motion with the individual depreciation rate of time as (negative) drift. We consider all individual depreciation rates constant in time and consider the average value of the individual depreciation rates of all people within the economy as the depreciation rate of human time (δ_T). Depreciation

[27] The discount rate (WACC) could also include the value of a tax-shield when funding with debt

of human time is like the concept of the risk-free rate r_f in corporate finance. These are the factors that drive the value of an economy which is expressed by formula (4.6) that relates the value of the average expected aggregated production of any given year *i+1* to the year before.

4.3 The zen of aggregated value

4.3.1 The value of an economy that does not invest equals Y_0/δ

We will now derive the equation that calculates the aggregated value of a closed economy denoted in units labour at present time (the reference level of labour productivity). Although all mathematics and formulas can be derived in a so-called continuous time-model, I will use discrete-time for the simple reason that it reduces the required level of the reader to just the four basic mathematical skills; adding up, subtracting, multiplying and dividing. Another advantage is that discrete-time models are more commonly used in corporate finance and therefore probably more familiar to anyone with a background in corporate finance.

We will assume no population growth. Appendix 7.1 includes population growth and appendix 7.2 derives equal results using a continuous-time domain.

Now let us consider the aggregated production Y_0 in year *0* of an economy with people that decide not (or no longer) to invest any of their time in capital goods. Instead they use the existing capital goods and exploit them. Hence all their aggregated production ($Y_0, Y_1, Y_2, \ldots$) is available for consumption goods. So at any given year *i* the aggregated consumption C_i equals the aggregated production Y_i. The downside is that the growth *g(s)* of this economy will be 0, whereas the annual depreciation of aggregated production due to decaying capital goods amounts to δ_0. To express the value of future aggregated production at present time (discounting) we need to divide the future aggregated production Y_i by $(1+\delta_T)$ for every year ahead (which equals *i* times for Y_i) and recall that $C_i=Y_i$.

$$Y_{i+1} = \frac{(1+\delta_0+g_r(s))Y_i}{(1+\delta_0+\delta_T)} \tag{4.6}$$

If we plug these numbers into equation (4.6) we derived earlier, we can see that for *i=0* this yields to $C_1= Y_1= Y_0/(1+\delta)= Y_0/(1+ \delta_0+ \delta_T)$. Similarly for *i=1* we can write $C_2=Y_2= Y_1/(1+\delta)= Y_0/(1+\delta)^2= Y_0/(1+ \delta_0+ \delta_T)^2$.

So, for any given year *i>0* we can write the value at present time of the aggregated consumption that equals aggregated production Y_i as a function of the aggregated production in our reference year Y_0.

$$C_i = Y_i = Y_0\left(\frac{1}{1+\delta}\right)^i = Y_0\left(\frac{1}{1+\delta_0+\delta_T}\right)^i \tag{4.7}$$

Now we define the value *V(0)* of an economy at the end of year *0* that does not or no longer invest any of its aggregated production in improving productivity of future labour (hence $s = 0$) as the sum of the values at present time of all expected aggregated consumption of years *i*, for *i=1* all the way up to infinity. This is expressed by formula (4.8).

$$V(0) \equiv \sum_{i=1}^{\infty} C_i = \sum_{i=1}^{\infty} Y_i \tag{4.8}$$

Please note that we start the summation at *i=1*. This is because we measure value at the beginning of year *1*, so year *0* has just past and has no value for humans as it lies in the past.

If we use formula 4.7 to denote Y_i by a function of Y_0 in (4.8) we get:

$$V(0) = \sum_{i=1}^{\infty} Y_0\left(\frac{1}{1+\delta}\right)^i \tag{4.9}$$

This can also be written as a sequence as follows*:*

$$V(0) = \frac{Y_0}{(1+\delta)} + \frac{Y_0}{(1+\delta)^2} + \frac{Y_0}{(1+\delta)^3} + \cdots + \frac{Y_0}{(1+\delta)^\infty} \tag{4.10}$$

We can rewrite this in the following form:

$$V(0) = \frac{Y_0}{(1+\delta)}\left[1 + \frac{1}{(1+\delta)} + \frac{1}{(1+\delta)^2} + \cdots + \frac{1}{(1+\delta)^\infty}\right] \quad (4.11)$$

Now this may look difficult, but in fact is a very simple formula if we use the following Taylor-polynomial[28]:

$$\sum_{i=0}^{\infty} x^i = 1 + x + x^2 + \cdots + x^\infty = \frac{1}{1-x}, \ \forall\, x \in (-1,1) \quad (4.12)$$

In words this means that if x is a number between -1 and 1, the sum of the indefinite polynomial ($1+x+x^2+x^3+\ldots$) all the way up to x^∞ equals 1 divided by 1 minus x.

If we define x as $1/(1+\delta)$, then we can write formula (4.11) as a Taylor-polynomial:

$$V_{s=0} = \frac{Y_0}{(1+\delta)}[1 + x + x^2 + \cdots + x^\infty], \text{ with } x = \frac{1}{(1+\delta)} \quad (4.13)$$

Since δ is always a positive fraction (percentage) it can be any number between 0 and 1. Therefore $x=1/(1+\delta)$ meets the criterium that it lies between -1 and 1. Formula (4.11) now can be rewritten as:

$$V(0) = \frac{Y_0}{(1+\delta)}\left(\frac{1}{1-x}\right) = \frac{Y_0}{(1+\delta)}\left(\frac{1}{1-\frac{1}{1+\delta}}\right) \quad (4.14)$$

If we multiply both the numerator and the denominator of the second term in equation (4.14) by $(1+\delta)$ we get our result (4.15).

$$V(0) = \frac{Y_0}{(1+\delta)}\left(\frac{(1+\delta)}{1+\delta-1}\right) = \frac{Y_0}{\delta} = \frac{Y_0}{(\delta_0+\delta_T)} \quad (4.15)$$

In words this means that the aggregated value at the end of year *0* of an economy that had an aggregated annual production Y_0 in the past year that decides not to invest any labour ($s=0$) in improving its future productivity of labour has a value of $Y_0/(\delta_0+\delta_T)$, wherein δ_0 is the expected annual percentage of loss of productivity due to decay of the capital goods in the economy (depreciation) and δ_T is the depreciation rate of human time that accounts for the fact that present time has more value to humans than future time. Now this result is very similar to the net present value of a company that generates an annual free cash flow ("FCF") available for its shareholders with no growth that is discounted with an annual discount rate of *r* [29], which reflects both the time value of money and the risk associated with the expected future cashflows.

$$Present\ Value = \frac{FCF}{r} \quad (4.16)$$

Formula (4.16) is a widely known and often used within corporate finance. The difference with formula (4.15) is that formula (4.15) includes depreciation of capital goods instead of a discount factor to account for risk, which is common in corporate finance. Secondly, formula (4.15) is denoted in productive labour hours in reference year *0* instead of currency.

4.3.2 The value of an economy that does invest in capital goods

Now suppose we are again at the end of year *0* (the beginning of year *1*) and imagine a similar economy with an aggregated annual production of Y_0 in year *0*. In this economy the people do decide to invest a fraction *s* (with $0 < s < 1$) of their aggregated production into capital goods every year from year 1 onwards. As a result, their economy (or aggregated production) is expected to grow with *g* every year from year 2 onwards ($i > 0$). Obviously, their aggregated consumption is a fraction *s* lower than would have been the case if the people did not invest in capital goods. Hence the aggregated consumption (*"C"),* which represents value to the population, in year 1 amounts to $(1-s)Y_1$, in year 2 the aggregated consumption is $(1-s)Y_2$, and so on. In any given year *i*, the aggregated consumption C_i equals $(1-s)Y_i$. The reward of this investment though is that the expected annual growth of the aggregated production will be *g* from year 2 onwards. Finally, we must account for depreciation

[28] J.H.J. Almering e.a., Analyse, Delftse Uitgevers Maatschappij B.V., zesde druk 1993, H10.6.5 d), pg 541
[29] Brealey Myers page 40. If we enter zero growth $(g=0)$ in the Key Value Driver formula 3.1 we get the same result since NOPLAT = FCF and WACC = r (discount rate or opportunity cost of capital).

δ_0 of the capital goods in this economy and the depreciation rate of human time δ_T, jointly referred to as $\delta=\delta_0+\delta_T$. If we use formula (4.6) and recall that $C_i=(1-s)Y_i$ and assume that growth will not occur in year *1*[30] we can denote the value of the expected aggregated consumption in year *1* at present time (C_1) as follows:

$$C_1 = (1-s)Y_1 = (1-s)\frac{Y_0}{(1+\delta)} = \frac{(1-s)Y_0}{(1+\delta_0+\delta_T)} \quad (4.17)$$

We need to value consumption now instead of aggregated production. This is because the people now decide to invest, so the aggregated production that is invested (sY_i) has no value to the people in year *i*. From year 2 onwards, the aggregated production is expected to grow and therefore the value of C_2 at present time will be:

$$C_2 = (1-s)Y_2 = (1-s)Y_1\frac{(1+g)}{(1+\delta)} = (1-s)Y_0\frac{(1+g)}{(1+\delta)^2}, with\ \delta = \delta_0 + \delta_T \quad (4.18)$$

In accordance with the equation (4.18) the present value of any given expected aggregated consumption C_i in year *i* will be:

$$C_i = (1-s)Y_i = (1-s)Y_0\frac{(1+g)^{i-1}}{(1+\delta)^i} \quad (4.19)$$

Now we define *V(s)* as the value of this economy at the end of year 0 that invests a fraction *s* of its aggregated production in capital goods (hence $0 < s < 1$) as the sum of the present values of all annual expected aggregated consumptions of years *i*, for *i* is 1 all the way up to infinity.

$$V(s) \equiv \sum_{i=1}^{\infty} C_i = \sum_{i=1}^{\infty}(1-s)Y_0\frac{(1+g)^{i-1}}{(1+\delta)^i} \quad (4.20)$$

This can be rewritten as a sequence:

$$V(s) \equiv \sum_{i=1}^{\infty} Y_i = Y_1 + Y_2 + \cdots + Y_\infty = \frac{(1-s)Y_0}{(1+\delta)} + (1-s)Y_0\frac{(1+g)}{(1+\delta)^2} + \cdots \quad (4.21)$$

Defining x as x=(1+g)/(1+ δ) and rewriting (4.21) we obtain:

$$V(s) = \frac{(1-s)Y_0}{(1+\delta)}[1 + x + x^2 \dots + x^3], with\ x = \left(\frac{1+g}{1+\delta}\right) \quad (4.22)$$

If we rewrite the Taylor-polynomial as [1/1-x] we and plug in $x=(1+g)/(1+\delta)$ we get the following:

$$V(s) = \frac{(1-s)Y_0}{(1+\delta)}\left[\frac{1}{1-x}\right] = \frac{(1-s)Y_0}{(1+\delta)}\left[\frac{1}{1-\left(\frac{1+g}{1+\delta}\right)}\right] \quad (4.23)$$

Please note that for this to be valid x can be any number between 1 and -1. Therefore, δ must be larger than *g*. Hence, we must always keep in mind that we can only use the results provided that the depreciation of human time δ_T exceeds real growth $g_r(s)$, in formula this is ***($g_r(s) < \delta_T$).***

Finally, we should multiply both the denominator and the numerator by $(1+\delta)$ to get our result:

$$V(s) = \frac{(1-s)Y_0}{(1+\delta)}\left[\frac{1+\delta}{1+\delta-(1+g)}\right] = \frac{(1-s)Y_0}{(\delta-g)} = \frac{(1-s)Y_0}{(\delta_0+\delta_T-g(s))} => V_s = \frac{(1-s)Y_0}{(\delta_T-g_r(s))}, if\ s > s_0 \quad (4.24)$$

This is the key value driver formula that calculates the value at the end of year 0 of an economy with a constant population and with an aggregated production Y_0 at year *0* that invests a fraction s of its aggregated production Y_i every year onwards from year *1* into capital goods, such that the aggregated production is expected to annually grow by *g* from year *2* onwards, with a depreciation rate that drives an annual productivity loss of the capital goods of rate δ_0 and accounts for the value of human time by means of the depreciation of human time δ_T, jointly referred to as $\delta = \delta_0 + \delta_T$.

[30] We disregard growth of year 1 (which would have been the case if the people were already investing a fraction *s* in year *0* and before. If we want to include this we should multiply our end-result with a factor $(1+g_r)$.

Now our final step in obtaining an equivalent of formula (4.1) is to rewrite *s* in formula (4.24). To do this, we define ROIC (Return On Invested Capital)[31] as the annual expected growth of any year *i* divided by the annual investment of any given year *i*:

$$ROIC = \frac{gY_i}{sY_i} = \frac{g}{s} \tag{4.25}$$

Formula (4.25) shows that ROIC will then be *g* divided by *s*. In analogy with corporate finance this ROIC calculates the annual benefit gY_i of investing a fraction of the aggregated production sY_i into capital goods instead of our aggregated consumption. If we rewrite (4.25) as a function of *s* we get a similar result as (4.1)[32]:

$$V(s) = \frac{Y_0\left(1-\frac{g}{ROIC}\right)}{(\delta-g)} \tag{4.26}$$

Appendix 7.1 and 7.2 show that in case we include annual population growth of *n* the formula's (4.24) and (4.26) look like the following:

$$V(s,n) = \frac{Y_0\left(1-\frac{g}{ROIC}\right)}{(\delta-g-n)} = \frac{Y_0(1-s)}{(\delta-g-n)} \tag{4.27}$$

Please note that equation (4.24) matches our result of equation (4.15) in case the economy does not invest *(s=0).* Now it would be interesting to see what the value is of an economy that invests at the replacement rate s_0 (with $0 < s_0 < 1$) such that the expected growth $g(s_0)$ equals the depreciation rate δ_0. As a result, their annual aggregated production is expected to remain constant since the annual depreciation is exactly offset by the expected annual growth. To see the value $V(s_0)$ of such an economy we need to enter $g(s_0) = \delta_0$ into formula (4.24).

$$V(s_0) = \frac{(1-s_0)Y_0}{(\delta_0+\delta_T-g(s_0))} = \frac{(1-s_0)Y_0}{(\delta_0+\delta_T-\delta_0)} = \frac{(1-s_0)Y_0}{\delta_T} \tag{4.28}$$

Formula (4.24) and alike is not meant to perfectly describe the value of an economy. Merely it is meant to get a better understanding of the dynamics that drive the value of an economy. Please refer to appendix 7.2 for the continuous time case that yields a similar result. I believe formula (4.24) is the solution of a neo-classical model like Solow-Swan, Cobb-Douglass or endogenous growth theory, except that it only has 1 production factor instead of 2 to 4, which makes the differential equations considerably easier to solve. To see this, please consider the parallels between Solow-Swan[33] and Appendix 7.2 "Value of an investing economy with population growth in continuous time".

4.4 Diminishing marginal returns and the optimal investment rate

When focusing on *g(s)* as a function of *s* the law of diminishing marginal returns comes into play. This law states that in all productive processes, adding more of one factor of production (for example adding capital by investing), while holding all others constant (in our case labour), will at some point yield lower incremental per-unit returns[34]. For example, any company that continues to invest in additional production capacity will have overcapacity which will drive price reductions and hence diminishing marginal returns. Adding a seventh lane to a highway may not yield as much traffic increase as the second lane once did, simply because there is no more traffic. Or leveraging on your workforce by investing in automation might at some point come to an end.

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[31] RONIC (Return on New Invested Capital) would be a better term since it expresses additional return divided by incremental capital invested.

[32] The difference that remains is that formula 4.1 uses the cash flow of year 1 ($FCF_{t=1}$) whereas formula (4.27) uses the production of year 0 (Y_0). The reason is that we did not assume growth in year 1. If we would have done this (implicitly assuming that the people already were investing a fraction *s* in the years before year 1) our result would have been real growth ($1+g_r$) in year 1, which would have increased the value of the economy by a factor ($1+g_r$). Because $(1+g_r)Y_0$ equals Y_1 we can replace Y_0 with Y_1 and obtain the equivalent of formula (4.1).

[33] https://en.wikipedia.org/wiki/Solow%E2%80%93Swan_model

[34] https://en.wikipedia.org/wiki/Diminishing_returns

All this also reflects that that maintenance investments generally have a higher return than for example fundamental innovation since it requires a low effort and secures the upside of the capital good.

Now this concept seems to make sense to our situation. We assumed reasonable behaviour of the population within the economy earlier, so we might expect that (1) they have their priorities straight when it comes to creating capital goods and (2) the population organises itself such that tasks are performed by individuals that are best qualified to do the job. This also holds for the fraction *s* of the productive time that is allocated to invest. We assume that at any given level of *s* the people that are occupied by investing (or the fraction of people's time that is used for investment) is optimal. This means for example that the population that is focusing on R&D is relatively constant over time and hence has gained skills, education and experience about their field of work. Or it could also mean that people have congruent careers. If they study medicine they are likely to become a doctor, if they work in Finance their professional education will largely be in that field too and people will likely not switch from one department or industry to a completely different department year on year during their careers. Summarising we can expect that (1) investments with high expected return have priority and (2) the pace of innovation (i.e gaining new knowledge) at a certain investment rate *s* is the best the people can do.

Therefore, the incremental growth *g(s)* will decrease as *s* increases. As *s* increases (1) the economy will invest in capital goods with lower expected return and (2) the level of competence of the people involved with investing decreases. The conclusion is that the law of diminishing marginal return applies to the *g(s)*-curve of economies.

To find a relationship between s and *g(s)* recall that at *s=0, g(s)* will be *0,* meaning that if the people do not invest any productive time in productivity improvement, they will not improve their productivity. Furthermore, the theoretical maximum level of *g(s)* occurs when everybody is full-time occupied with investing. So we denote this level of growth with g_{max}. This occurs at *s=1,* so $g(s=1)=g_{max}$. Finally, due to the concept of diminishing marginal returns it is obvious that the gradient (or slope) of *g*-curve will be high (steep) just above s=0 and gradually flatten when approaching g_{max} at *s=1*. These characteristics are best described when *g(s)* has the form of a power-law $A*s^\gamma$ with γ between *0* and *1* ($0 < \gamma < 1$). We can now describe *g(s)* as follows:

$$g(s) = g_{max}(s)^\gamma, 0 \le s \le 1 \; and \; 0 < \gamma < 1 \quad (4.29)$$

To get an idea of how this might look we plotted this function in figure 4.1 for g_{max}= *5,5% and* γ=0,15, which are just arbitrarily chosen numbers.

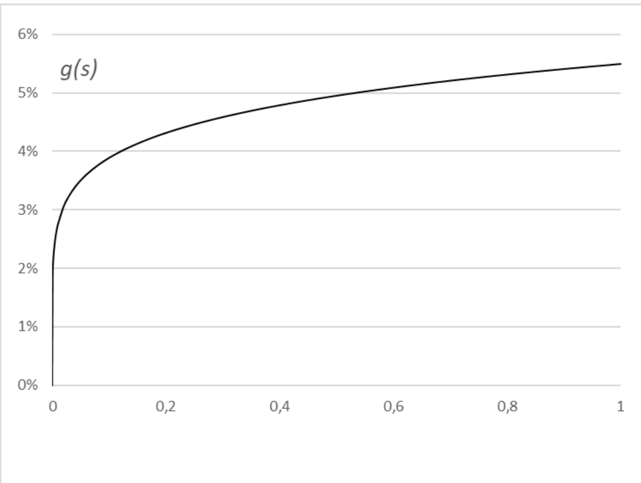

Figure 4.1 $g(s) = g_{max} * s^\gamma$, *with* g_{max} = *5,5% and* γ = *0,15*

Let's assume the depreciation δ_0 amounts to 4%. This is again an arbitrarily chosen percentage, the theoretical value is the aggregated weighted average of all capital goods in the economy. This includes tangible goods and intangible goods like human knowledge. The lifetime of tangible goods varies from roughly one year to hundreds of years (buildings and bridges) and human related capital goods like social networks and expertise are proportional to careers. That is a large bandwidth. Piketty states that annual depreciation is roughly 10%

for most countries[35]. Considering that this according to his definition excludes human-related capital goods (like knowledge and networks), the 4% we assume might not be far off. We can now use (4.29) to plot the real growth (on top of δ_0) in figure (4.2).

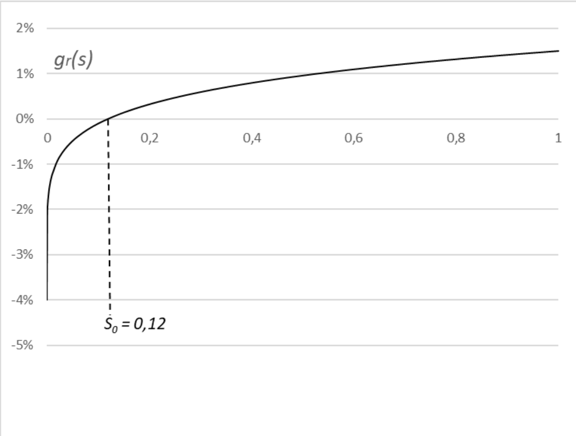

Figure 4.2 $g_r(s) = g_{max} * s^{\gamma} - \delta_0$*, with* g_{max} *= 5,5%,* γ *= 0,15 and* δ_0*=4%*

In this case s_0 is 0.12. Now we have an equation for *g(s)* and $g_r(s)$ we can plug this into equation (4.29) as follows (assuming no population growth). We assumed the depreciation rate of human time *(*δ_T*)* amounts to 2%

$$M = \frac{V_S}{Y_0} = \frac{(1-s)}{(\delta_0+\delta_T-g(s))} = \frac{(1-s)}{(\delta_0+\delta_T-g_{max}s^{\gamma})}, with\ 0 < s < 1\ and\ 0 < \gamma < 1 \qquad (4.30)$$

Obviously the larger the fraction *s* that the economy invests in capital goods, the larger the growth rate *g(s)* will be, but the lower the remaining fraction of the aggregated production that will be available for consumption goods which has a negative effect on the value of an economy. In other words, if all people within an economy spend all their time in studying books and building bridges and robots, there is nobody left to produce consumption goods like food to keep the population alive. This is what the *(1-s)* describes in equations (4.30), (4.29) and (4.24) and alike.

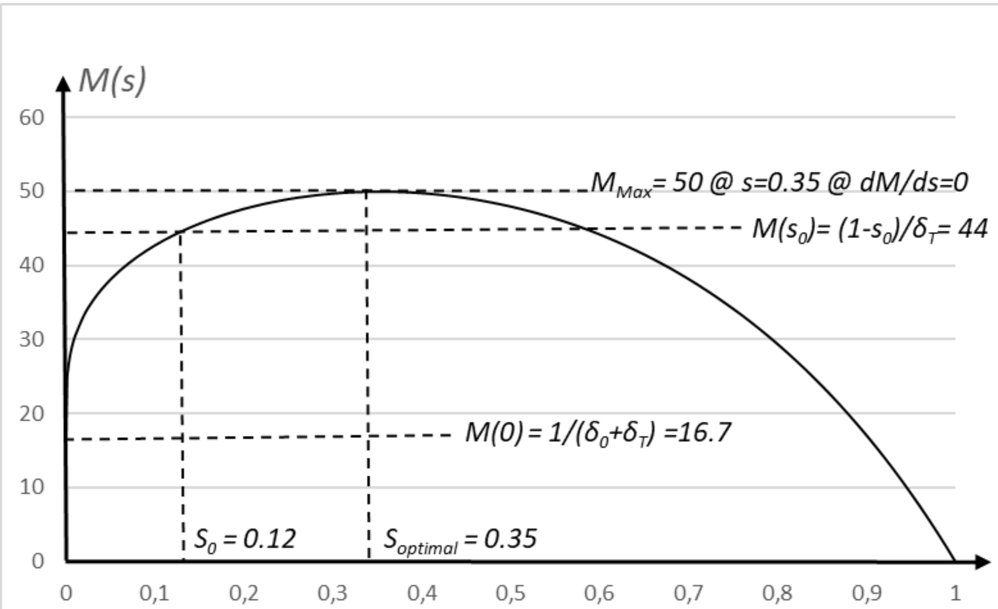

Figure 4.3 M(s) as a function of s, with gmax= 5.5%, γ *= 0,15,* δ_0*=4% and* δ_T*=2%*

[35] Piketty page 59

In figure 4.3 we plotted *M(s)* using the same numbers for g_{max}= 5.5%, δ_0=4% and δ_T = 2%. We also visualised the values of non-investing and replacement-rate investing economies respectively at (1/$\delta_0+\delta_T$) and (1-s_0)/δ_T respectively.

In line with what we expected the value of an economy becomes *0* if s approaches to *1.* This makes sense, because when all available productive time (labour) is allocated to the production of capital goods, there is nothing left for consumption goods and people die so the economy has no value anymore.

Finally, what figure 4.3 tells us is that there is an optimal investment rate that maximises the value of an economy at V_{max}. This optimum can be found by differentiating equation (4.30) with respect to *s* and solve this for *dM/ds* = 0.

$$\frac{dM(s)}{ds} = \frac{d}{ds}\left(\frac{1-s}{\delta_0+\delta_T-g(s)}\right) = \frac{d}{ds}\left(\frac{1-s}{\delta_0+\delta_T- g_{max}s^{\gamma}}\right) = 0 \qquad (4.31)$$

In our example, the *dM/ds=0* at *s=0.35* such that the maximum value of the economy amounts to 50 times Y_0.

Obviously, formula 4.30 is not exact. It merely provides an indication of the real curve. In case the people take rational decisions, I believe it should be a concave curve though. Also it might help to see why emerging economies grow so much faster than mature economies – the investment opportunities are proven technology (copied from mature economies) and the growth prospects are enormous (like building infrastructure). Therefore, the g_{max} is (temporarily) much larger and γ is closer to 1, that jointly reflect an abundance of high-return investment opportunities that will gradually decrease as the economy matures.

4.5 Aggregated time accounting and quality of life

Please recall that until now we assumed that in every future year the people continue to work similar hours when deriving the value of their economy. All labour productivity increases are then reinvested in further labour productivity growth and consumption (pro rate parte if *s* remains constant). In analogy with asset pricing this assumes a self-financing process which means that all dividends are reinvested. Obviously, the people can also choose to enjoy more leisure time at the cost of future consumption. This section will derive the time statements (i.e. the equivalent of financial statements) of a closed economy as defined earlier. The concept is exactly like financial accounting. In financial accounting the profit & loss account ("P&L") means to tell us the difference between the value of the production output of a company (revenues) and the value of the resources it required to do so (costs). These costs can be operational expenses (present time use of resources) and depreciation (decay of prior investments in production equipment or alike). The cashflow statement records all actual payments in a certain period and shows cash inflows minus expenditures. Operational expenditures (opex) are accounted for in the P&L, but capital expenditures (capex) are investments and are capitalised on the balance sheet and accounted for in future's P&L by depreciating on the capitalised expenditures (i.e. fixed assets). This way, the differences between the P&L and the cash flow statement are accounted for on the company's balance sheet. Normally there also exist differences in short term liabilities and assets that show up in the cashflow statement as working capital mutations, but these are largely neglected in this section. Applying these principles of financial accounting on time accounting yields the following table that explains the meaning of the commonly used line items of the P&L, cashflow statement and balance sheet regarding time accounting[36].

[36] We use corporate finance terms (like NOPLAT and free cash flow) instead of financial accounting terms. Please refer to "Koller e.a." for the meaning of these terms in financial accounting

AGGREGATED PROFIT & LOSS ACCOUNT			
Financial statement line item		**Operator**	**Meaning regarding time accounting**
	Total aggregated production		Total production of useful goods and services in period *i*
ΔWC_i	Changes in working capital		Any changes in aggregated stocks of finished and semi-finished goods (or extracted raw materials and work in progress). If the people produce more than they consume stocks increase and vice versa. Since we cannot save time debtors and payables do not exist, so changes in working capital are equal to stock changes. For simplicity we ignore changes in stock, which seems fair on the long run (expiry or waste of stocks is considered consumption)
$Sales_i$	Sales/ $Revenues_i$ *(Y_i)*		Total consumption of all useful time in period *i* by consuming goods and services
$Opex_i$	Operational expenses ($opex_i$)	-/-	Consumption of present time (in period *i*)
$EBITDA_i$	Earnings Before Interest, Taxes, Depreciation And Amortisation	= A-B	We disregard time debt and amortisation (to the extent it exists)
D_i	$Depreciation_i$	-/-	Consumption of historic useful time (depreciation of useful time that was capitalised prior to period *i*)
$NOPLAT_i$	Net Operating Profit Less Adjusted taxes	EBITDA -/- D	Net result of the aggregated production of useful time minus the aggregated consumption of useful time. Time taxes do not exist in nature

Table 4.1 Meaning of profit & loss account line items regarding aggregated time accounting

AGGREGATED CASH FLOW STATEMENT (INDIRECT METHOD)			
Financial statement line item		**Operator**	**Meaning regarding time accounting**
$EBITDA_i$	EBITDA		
ΔWC_i	Working capital mutations	+ or -/-	This would be the difference of production of goods minus consumption which equals the increase or decrease of aggregated stocks. We ignore this which implies that annual production of consumption goods are always equal to consumption.
$Capex_i$	Capital expenditures	-/-	Time invested in the creation of capital goods (including education of people etcetera) that is capitalised on the aggregated balance sheet
FCF_i	Free Cash Flow	=	The remaining aggregated available useful time (labour) that was neither spent on creation of consumer goods nor on creation of capital goods and apparently was spent as free time (leisure in case of our well-functioning economy) instead of labour.
DIV_i	Dividend	=	In time accounting the equivalent of Free Cash Flow equals dividend payments since time cannot be saved. Unspent labour therefore automatically is free time

Table 4.2 Meaning of cashflow statement line items regarding aggregated time accounting

AGGREGATED BALANCE SHEET			
Financial statement line item		**Operator**	**Meaning regarding time accounting**
Total assets			
TFA_{i-1}	Total Fixed Asset	+	Total capitalised historic labour at the beginning of period *i*
$Capex_i$		+	Capitalised labour in period *i* is added to the total fixed assets (based on the historic costs method)
Dep_i	$Depreciation_i$	-/-	Decay of fixed assets during period *i* is subtracted from the fixed assets
TFA_i	Total Fixed Asset	=	Which equals the total capitalised historic labour at the end of period *i*
WC_i		+	Working capital mutations (i.e. stock movements of consumption products and semi-finished goods) are neglected but otherwise should be accounted for at the balance sheet
TA_i	Total Assets	=	Since we disregard working capital the total assets equal total fixed assets which amounts to the total aggregated value of the capital goods in the economy (as described earlier)
Total liabilities			
E_{i-1}		+	Equity at beginning of period *i*
$NOPLAT_i$		+	The net profit of year *i* that is added to the reserves of the aggregated economy…
DIV_i		-/-	…subtracted by all dividend payments in period i…
E_i		=	…equals the total value of the equity at the end of year i.
TL_i	Total liabilities	=	Debt (both short-term and long-term are disregarded) hence the total liabilities equal the aggregated equity of the closed economy

Table 4.3 Meaning of balance sheet line items regarding aggregated time accounting

Now we understand the equivalent meaning of financials statements regarding time accounting we can draft the aggregated time statements of a well-functioning closed economy as discussed in previous sections. The meaning of the symbols used remains unchanged.

Please find below the aggregated time statements of a closed well-functioning economy for years *0* to years *i*. Assets are capitalised based on the "historic costs method".

Aggregated Profit & loss account				
Line item	**Operator**	**Year 0**	**Year 1**	**Year i**
Revenues	+	Y_0	$Y_0(1+g)$	$Y_0(1+g)^i$
Opex	-/-	$(1-s)Y_0$	$(1-s)Y_0(1+g)$	$(1-s)Y_0(1+g)^i$
EBITDA	=	sY_0	$sY_0(1+g)$	$sY_0(1+g)^i$
Depreciation	-/-	$\delta_0 Y_0$	$\delta_0 Y_0(1+g)$	$\delta_0 Y_0(1+g)^i$
NOPLAT	=	$(s-\delta_0)Y_0$	$(s-\delta_0)Y_0(1+g)$	$(s-\delta_0)Y_0(1+g)^i$

Table 4.4 Aggregated profit & loss Account denoted in the amount of useful produced time in the present year (aggregated production Y_0)

Aggregated Time flow statement				
Line item	**Operator**	**Year 0**	**Year 1**	**Year i**
EBITDA	+	sY_0	$sY_0(1+g)$	$sY_0(1+g)^i$
WC Adjustments	-/-	0	0	0
Capex	-/-	φsY_0	$\varphi sY_0(1+g)$	$\varphi sY_0(1+g)^i$
Free Time Flow (FTF)	=	$(1-\varphi)sY_0$	$(1-\varphi)sY_0(1+g)$	$(1-\varphi)sY_0(1+g)^i$

Table 4.5 Aggregated time flow statement denoted in the amount of useful produced time in the present year (aggregated production Y_0)

Aggregated Balance sheet				
Line item	**Operator**	**Year 0**	**Year 1**	**Year i**
Assets				
Capital goods at the beginning of period	+	0	$(\varphi s-\delta_0)Y_0$	$(\varphi s-\delta_0)Y_0 +$ $(\varphi s-\delta_0)Y_0(1+g)$
Investments (capex)	+	φsY_0	$\varphi sY_0(1+g)$	$\varphi sY_0(1+g)^i$
Depreciation	-/-	$\delta_0 Y_0$	$\delta_0 Y_0(1+g)$	$\delta_0 Y_0(1+g)^i$
Capital goods at the end of period	=	$(\varphi s-\delta_0)Y_0$	$(\varphi s-\delta_0)Y_0 +$ $(\varphi s-\delta_0)Y_0(1+g)$	$(\varphi s-\delta_0)Y_0 +$ $(\varphi s-\delta_0)Y_0(1+g)$ $+\dots+$ $(\varphi s-\delta_0)Y_0(1+g)^i$
Liabilities				
Equity at the beginning of period	+	0	$(\varphi s-\delta_0)Y_0$	$(\varphi s-\delta_0)Y_0 +\dots+$ $(\varphi s-\delta_0)Y_0(1+g)^{i-1}$
NOPLAT	+	$(s-\delta_0)Y_0$	$(s-\delta_0)Y_0(1+g)$	$(s-\delta_0)Y_0(1+g)^i$
Dividend (FTF)	-/-	$(1-\varphi)sY_0$	$(1-\varphi)sY_0(1+g)$	$(1-\varphi)sY_0(1+g)^i$
Equity at the end of period	=	$(\varphi s-\delta_0)Y_0$	$(\varphi s-\delta_0)Y_0 +$ $(\varphi s-\delta_0)Y_0(1+g)$	$(\varphi s-\delta_0)Y_0 +$ $(\varphi s-\delta_0)Y_0(1+g)$ $+\dots+$ $(\varphi s-\delta_0)Y_0(1+g)^i$

Table 4.6 Aggregated balance sheet denoted in the amount of useful produced time in the present year (aggregated production Y_0)

In the statements above φ is called the investment rate, which is defined as the fraction of EBITDA (sY_i) that is reinvested in capital goods. It is the equivalent of the investment rate as described by Koller e.a. (page 61), although that investment rate is related to NOPLAT instead of EBITDA.

We can now define the annual increase (or decrease) in **quality of life** in year *i* (based on historic costs) as the amount of labour investment in capital goods minus the depreciation of capital goods $(s-\delta)Y_i$ during the period *i*.

This equals **NOPLAT**$_i$ = $(s-\delta_0)Y_0(1+g)^i$ which is the equivalent of the net profit of an unlevered company in financial accounting. The time dividend or free time flow $DIV_i = FTF_i = (1-\varphi)sY_0(1+g)^i$ is the share of the increase in quality of life that was spent on increased future leisure time and the remaining part $(\varphi s-\delta_0)Y_0(1+g)^i$ is reinvested in increased future consumption which is accounted for by adding this to the equity reserves of the economy.

Please remember that we derived the value of an economy in formulas (4.24) and (4.30) by assuming all productivity increases were reinvested in the economy (self-financing process). We did not take leisure time into account. Obviously, once people enjoy time dividend this impacts the growth of future labour productivity increase. Therefore, we should account for this by including the fraction φ of available labour sY_i that is used for creation of capital goods. If we adjust formulas (4.24) and (4.30) we get formulas (4.32) and (4.33):

$$V_s = \frac{(1-\varphi s)Y_0}{(\delta_0+\delta_T-g(\varphi s))} \Rightarrow V_s = \frac{(1-\varphi s)Y_0}{(\delta_T-g_r(\varphi s))}, if\ s > s_0 \qquad (4.32)$$

$$M = \frac{V_S}{Y_0} = \frac{(1-\varphi s)}{(\delta_0+\delta_T-g(\varphi s))} = \frac{(1-\varphi s)}{(\delta_0+\delta_T-g_{max}(\varphi s)^{\gamma})}, with\ 0 < s < 1, 0 < \varphi < 1\ and\ \ 0 < \gamma < 1 \quad (4.33)$$

Note that if φ equals 1 for all years than all labour available for creation of capital goods is reinvested in the economy (self-financing process) which would yield values as denoted by formulas (4.24) and (4.30). If φ is smaller than 1, formulas (4.32) and (4.33) describe the value of the quality of life due to consumption (at fair value instead of historic costs) and disregard the value of time dividends. The value of time dividends is simply obtained by subtracting formula (4.32) from formula (4.24), which equals the summation of (4.32) when φ is replaced by (1-φ). This describes a series of free time flows valued at their future level of quality of life compared to present time discounted by the depreciation rate of human time.

The annual aggregated production $Y_i = A_i L_i$ can be denoted in Labour *(L_i)* which is a function of population growth and dividend rate *(1-φ)* to account for adjustments in the annual working hours multiplied by a production efficiency function (A_i) that is driven by the investment rate *s* and the labour productivity growth. This way we could monitor efficiency gains, leisure time increases and population growth separately.

Summarising chapter 4, we can now conclude that people in a well-functioning economy that invest part of their labour supply in the creation and maintenance of capital goods increase their future quality of life. They can freely decide whether they spend the annual increase in quality of life on (1) more leisure time or (2) more future consumption.

PART 2 MONEY TO SHARE

5 Money to share

5.1 Introduction and scope

In chapter 4 we investigated how people jointly create value in a closed well-functioning economy, which we defined as an economy wherein (1) workers always do the best they can, (2) their cooperation is always optimal and (3) they always take rational organisational decisions when allocating jobs to people and when setting priorities. We ignored the distribution side of this value and ignored any malfunctions of a real economy. We said we would deal with that later. Well, now is the time and the concept of money comes into play again. In this chapter we will investigate how we currently register value (financial accounting) and divide this between ourselves (financial industry). We will do this by consolidating the aggregated financial statements of the economy which we can use to derive budget constraints.

Paragraph 5.2 relates time valuation to the generally accepted principles of asset pricing. Section 5.3 does the same by relating some of our current economical thinking to time accounting. Paragraph 5.4 discusses how I believe financial accounting and macroeconomics are related by consolidating all businesses that operate in what we will call a closed truly capitalistic economy. This will give us the tools we need to observe and analyse the financial system we are currently operating in western economies.

Paragraph 5.5 uses the consolidated financial statements of a closed truly capitalistic economy to derive budget constraints. These will help us better understand the instability and the dynamics of our financial system. The last part relates all this content to neo-classical growth theory (Solow).

The final paragraph 5.6 decomposes the aggregated budget constraint to understand how inequality in a closed truly capitalistic economy is increasing.

To support the content of part 2 of this book it comes with a spreadsheet that models a truly capitalistic closed economy with a fractional reserve banking system based on the mathematics in this chapter. Readers can use the spreadsheet model to verify the content of this chapter and adjust input variables to learn. The model allows for annual adjustment of all input variables and projects the consolidated financial statements of all households, governments, financial institutions and companies for the next 500 years. Also, it provides the debt characteristics and the inequality characteristics of the economy. The spreadsheets models tax flows like corporate income tax, dividend tax and labour tax, but disregards capital taxes and sales tax (VAT).

5.2 How time valuation relates to asset pricing

5.2.1 What does the generally accepted theory say about valuation and asset pricing?

First let me briefly describe our currently generally accepted theory of valuation and asset pricing. Anyone familiar with corporate finance and asset pricing might want to skip this section.

The first thing you might want to know about valuation (which by the way we already touched upon in chapter 4) is that the theoretical value of an asset (like bonds and stocks) equals the net present value of the expected future cashflows that the asset will generate in the future. This is a summation from next year's expected free cash flow to eternity, which all should be discounted at an appropriate discount rate. If both the discount rate and expected annual growth of the free cash flows remain constant over time the solution to the summation equals formula 4.1 taken from section 4.1 (or an equivalent form). The discount rate (WACC) is determined by (1) the risk-free rate which is the annual return on a riskless asset (also known as the "time value of money") added with (2) a premium that the owner of the asset demands for the risk associated with the investment[37]

$$Value = \frac{Free\ Cash\ Flow_{i=1}}{WACC - g} \qquad (4.1)$$

[37] Another value driver is the capital structure. Debt financing results in a reduced tax burden ("taxshield") that increases the cashflows available for debt- and equity holders. This is neglected here as it is purely financial engineering.

The second thing you might want to know about corporate finance is the meaning of the word “risk”. When investors (or other people familiar with corporate finance) talk about “risk” related to equity they in fact mean what everybody else on the planet would call “uncertainty”. Indeed, actual returns of the investment may turn out to be disappointing, but provided that the expected return is reasonable to expect (which is the investor’s job to assess), the chances on an outperforming return are equally likely. The expected return of an investor that invests in a risky asset is in fact the expected return. It is not like a bank that provides loans. Banks charge higher interest on risky loans. That I believe is a sensible thing to do because for banks risky loans indeed are risky – there is zero chance that any borrower would voluntarily repay more than the principle amount, but there is a chance the borrower defaults on its repayment obligations. For the bank the expected repayment is lower than what they provided which should be accounted for in the interest they charge.

Now we defined investors’ risk (i.e. uncertainty) we can propose the main tenet of corporate finance. Investors demand an additional return proportional to the risk exposure of the asset (company) they invest in, which thus essentially is exposure to uncertainty and not downside risk. This additional return on top of the risk-free rate is called the **equity premium**[38]. The risk-free rate as we know it today was previously discussed in chapter 4 and denoted by r_f.

To make the concept of the equity premium some more tangible, imagine you invite two investors to playing a fair coin-tossing game (“head-or-tails”) against yourselves. In corporate finance such a fair game with an expected gain of *0* is called a Martingale process[39]. The investor that chooses “tail” will receive USD 1,= from you for every throw that results in “tail” and has to pay USD 1,= to you if it is “heads”. For the other investor, the returns are the exact opposite. Since the coin is unbiased the chances on both “heads” and “tails” are 50% each for every throw, so the expected return is USD 0,= for both investors. And for a fact you will know that you will gain nor lose any money since every throw you will receive and pay USD 1,=. Hence you are exposed to the so-called risk-free rate, which in this example is 0%. Risk-free in this case means that you are certain you will not gain nor lose any money by participating in the game (no uncertainty). At least that is what you might think. Now, both the tail-investor and the heads-investor according to their perception on risk and return will demand an additional expected return as compensation for their exposure to uncertainty as they play the game. This in fact means that they value participating in the game as a liability. If you want them to play the game, you will have to compensate them for the possibility that they do not walk out of this game with exactly USD 0,=. So, let’s say they demand 2 cents (2% fee) for every throw of the dice to get them to play. Once the game is finished either investor might have gained or lost money, but for sure you know that you lost 4 cents per throw. Suddenly, it seems that the risk-free rate of return now is minus 4% and the additional required return to compensate for the uncertainty of the investors is 2% each. The 2% fee investors charge is like the equity premium in real capital markets. In real capital markets this is less visible though, because the expected returns of the investors and the risk-free-rate will likely all be positive, so nobody is losing money. Short-term governmental bonds (T-Bills) are often considered to be closest to the risk-free rate. For that reason, the interest on T-bills is often used as the best proxy for the risk-free rate of return.

Now the question is “who pays the 2% fees to each investor” in the real world? And the answer is simple: “the companies they invest in do”. This can be its customers, suppliers, employees, banks, anyone. One thing we know for sure though; it’s a zero-sum game, so all return that flows to investors is not flowing to someone else. To see how this might work in real life, imagine a company managed by executives that support the idea that the investors are entitled to an equity premium. This would impact all their decisions and hence will be accounted for in all investment decisions of the firm and all negotiations with customers, suppliers and employees about their terms. Free-market advocates would then argue that if this equity premium that the company accounts for indeed is overcharging investor returns at the cost of the other stakeholders, all other stakeholders could easily choose a competing company to work with. Employees are free to look for another job, suppliers can supply competitors that pay better prices and the customers can buy cheaper somewhere

[38] The premium for bearing risks is also commonly referred to as **risk premium**. I will use equity premium to emphasize that I explicitly refer to uncertainty (variance) and exclude the risk exposure to fixed-income securities (like bonds)
[39] For an exact definition and its application please refer to Back page 189

else. Now here is the problem – virtually every company in the world accounts for the equity premium, so there is no alternative than to accept that investors charge equity premiums.

What does the theory say about risk and return?

Since everybody seems to have been adopting this tenet of the equity premium for decades, let's find out what their rationale is.

In "Asset Pricing and Portfolio Choice Theory", a book that is intended for the introductory finance Ph.D. course in asset pricing theory, the author K.E. Back states in the introduction of the book[40]:

"The basic tenet of asset pricing theory is that securities that have higher risks should have higher average returns to compensate investors for bearing their risks."

Authors Brealey, Meyers and Allen state in their introduction of the risk section of their book "Principles of Corporate Finance", which is widely used by and primarily written for financial managers of companies that:[41]

"Most investors are not adrenaline junkies; they don't enjoy taking risks. Therefore, they require a higher expected return from risky investments. Companies recognize this in their capital budgeting decisions. An investment in a risky new project adds value only if the expected return is higher than investors could expect from an equally risky investment in the capital market."

In both books the proportional relation between risk and return is adopted as a tenet. The content of both books is about what this relationship should look like and how it impacts capital markets. The relationship itself is not argued or reconsidered.

Generally, the theory of asset pricing builds upon individuals that aim to optimise their utility- or wealth function *u(w).* This is a relationship that expresses utility (wealth) of an individual driven by (1) return on her (initial) wealth, (2) labour income less (3) consumption. Then the theory assumes that the individual wishes to maximise her utility function over a certain amount of time (which represents her lifetime or career), considering a certain risk-averse relationship that is applicable to the individual.

An individual is considered risk-averse if she would prefer to avoid a fair bet (like the coin-tossing game we just discussed above) over a guaranteed value equal to the expected value of the bet. The amount of risk-aversion is then related to the amount she is willing to pay to avoid the fair bet, which essentially is the "equity premium". Asset pricing theory describes a lot of different risk-aversion relationships that can be applicable to individual investors. They generally assume decreasing average risk aversion (DARA). This means that the risk-aversion of an individual decreases as her wealth increases. In other words - the richer she is, the lower the equity premium is she will demand to accept a fair bet. Mathematically this is mostly presented as second-order risk-aversion, which means that the risk-aversion is related to the variance σ^2 of a stochastic process as opposed to first-order risk aversion that relates risk-aversion to standard deviation σ.

Asset prices (for example company stock prices) are presented as stochastic discount factors with a mean expected return and a variance that defines the volatility. The higher the risk (volatility), the higher the expected return that investors require, the lower the stock price will be.

With the utility function defined and asset prices presented as stochastic discount factors the next step is to create portfolio's (baskets) of investments to optimise the risk-return balance of the investor. One can imagine that a basket of investments with stock prices that move independently from each other (they are not fully correlated) reduces the volatility of the portfolio. The expected portfolio return of this basket of investments on the other hand is the weighted average return of the various investments, regardless the volatility of the investments in the basket. So, investing in a portfolio (basket) of stocks reduces volatility (risk), but does not impact expected returns for the investor. This is called spreading risks. Harry Markovitz mathematically proved in the 1950's that for any given expected portfolio return there exists a portfolio of investments such that the

[40] Back page xviii
[41] Brealey Meyers page 145

risk is minimised. Portfolio's that have this property are called efficient portfolios (or frontier portfolio's). These portfolios are all on the curved line denoted by *T* in figure 5.1.

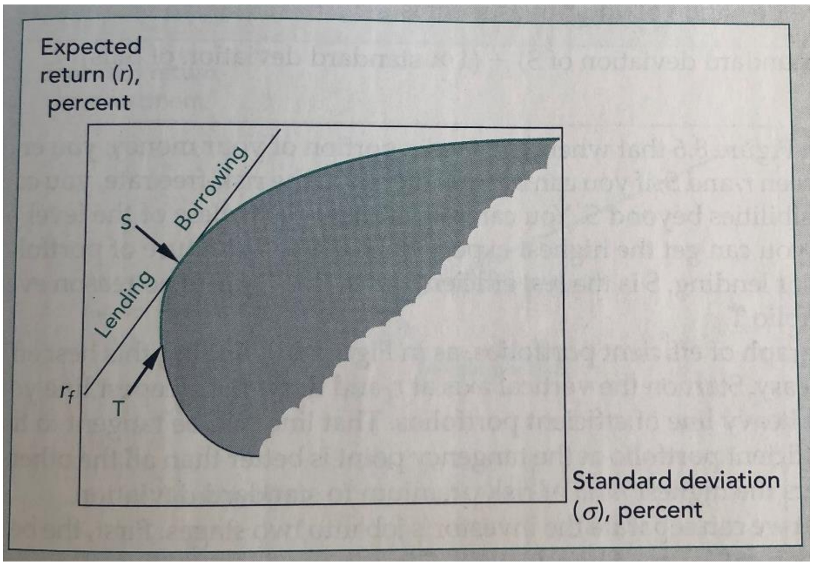

Figure 5.1 taken from Brealey Myers page 187

Please note the straight line from the risk-free rate r_f at zero variance tangent to the efficient portfolio curve *(T)*. This line represents yet another possibility. By lending or borrowing at the risk-free rate and investing in the efficient portfolio at point S (which is called the tangency portfolio), an investor can construct an efficient portfolio with any given expected return along this line. This is called "two-fund spanning". The principle of using debt to increase the expected return of an investment at the cost of increasing risk is called leverage and is often used by Private Equity investors (and by leveraged companies). We need to bear in mind though that hardly anyone (except maybe a few well-financed governments representing strong economies) can borrow money at the risk-free rate.

In the mid 1960's Sharp, Lintner and Treynor used Markovitz' portfolio choice theory to derive the so-called Capital Asset Pricing model (CAPM). This model still today is the most widely used model in corporate finance globally to determine required returns. Sharp e.a. assume that there are two types of risk, which are market risk and asset (company) specific risk. In line with portfolio choice theory they argue, investors can create a market portfolio that contains all risky assets (shares of companies) that are available in the market such that all specific risks of all these assets are eliminated leaving the investor exposed to just the undiversifiable market risk (which reflects economic cycles). The CAPM then states that the expected risk premium of any given single risky asset should be proportional to its correlation with the variance (volatility) of the market portfolio. The correlation of a stock price movement with the market portfolio is called *beta.* It can then be calculated that in well-functioning capital markets (i.e. without arbitrage opportunities) all expected risk premiums of single risky assets lie along a line that connects the risk-free rate and the market portfolio. This line is called the Security Market Line or Capital Market Line.

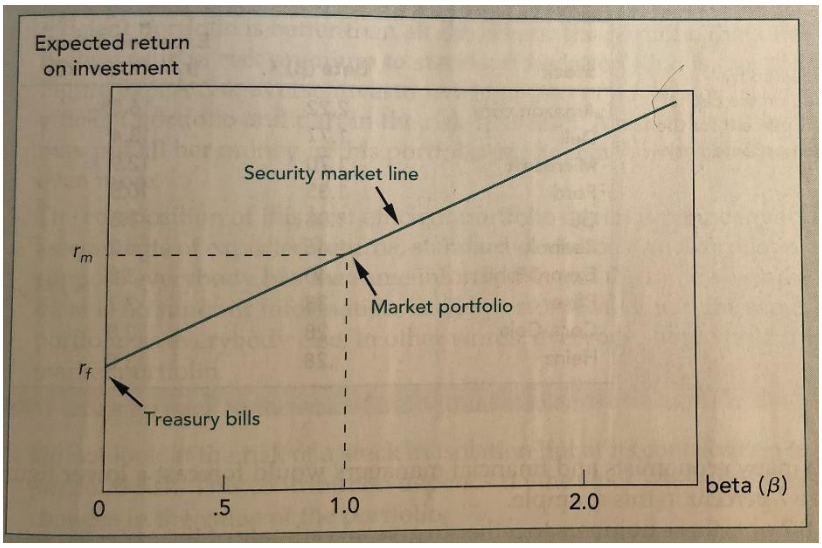

Figure 5.2 taken from Brealey Myers page 189

The market return r_m is the return of the market portfolio which is the risk-free rate added by the market premium. The market premium is the equity premium that investors demand on top of the risk-free rate for bearing the risk of a market portfolio. This is all visualised in figure 5.2 taken from Brealey Myers page 189. The expected return (r) of any given single risky asset in the market is now completely determined by its *beta,* the risk-free rate r_f and the expected market return r_m. This is expressed in formula (5.1). The right side of the equation is the equity premium of the asset.

$$r - r_f = beta * (r_m - r_f) \qquad (5.1)$$

Obviously, the *beta* of the market portfolio itself is *1*, which implies that it is 100% correlated with its own volatility, being the market volatility. Stocks with beta's larger than *1* tend to overrespond to economic cycles and are called cyclical assets. For assets with a beta lower than *1* (or even negative), you will see that the equity premium is lower than the market premium. The rationale is that these assets are valuable to investors because of their contribution to reducing the risks of a well-spread portfolio. So, according to the CAPM assets should not be valued based on their volatility, but on their correlation with the market volatility. This is because every investor can hedge all specific risks of all individual assets by investing in the market portfolio, so specific risk (the assets' price volatility) are eliminated due to investors looking for arbitrage opportunities in well-functioning markets. The premium for bearing generic (undiversifiable market) volatility is the market premium.

The CAPM is a so-called single-factor model, wherein the market correlation (beta) is the factor that drives the equity premium. There are several other factor-models available that include other factors like company size, recent stock movement, book-to-value ratio, aggregated consumption that are assumed to drive equity premiums.

Liquidity and defaulting

Finally, we should keep in the back of our minds that there are two other factors that might influence the equity premium. These are the value of liquidity and the risk of defaulting. With the value of liquidity, I mean that investors generally assign value to having the option to sell their assets instantly at a fair value. Large cap stocks (stocks of companies with a large market capitalisation) are usually easier to sell and at larger volumes without impacting trade prices than small cap stocks are. This could result in a price discount on small cap stocks traded at the capital markets. In this regard investors could also appreciate low volatility over high volatility, because chances that stocks with low volatility trade (well) below the fair value is smaller in case investors unexpectedly are required to sell immediately. With risk on default I mean that there is a chance that a company does not meet its obligations and files for bankruptcy, or the stock is taken by financing banks. This way, the stockholders lose their investment forever. The risk on default has a certain impact on the volatility curves such that it is not fully symmetric around the expected return (zero chance below zero return, uncapped upside). This could impact the risk premium. The risk on defaulting increases as the financial leverage (amount of debt relative to the company's profitability) increases. Although liquidity and defaulting might impact the volatility such that is downside risk of a stock is slightly higher than the upside we could also assume that the market portfolio includes banks and other institutions that hold counter positions such that they would benefit from defaulting companies and offset any possible negative bias due to liquidity and defaulting within the market portfolio.

That sounds reasonable too, provided that investors are risk-averse. But should they be?

If you take the perspective of an individual with an income from labour and some savings who aims to maximise her wealth during a certain limited expected timeframe (her life), it indeed does not seem unreasonable to assume that she is risk-averse, which is the tenet that creates the equity premium in financial markets. However, I doubt whether this individual represents the investors that dominate today's capital markets. My guess would be that today's capital markets are dominated by professional investors that invest for the account of pension funds, insurance companies, asset management firms and alike. These professional investors differ from individuals since they have (or should have) a going concern principle, which allows them to have an indefinite view on investing for most of their budgets. They do not retire at some point in the next 40 years like individuals do. Secondly, their budgets are large enough to create market portfolio's that enable them to spread risk and reduce volatility. Thirdly, these investors are professionals that should be able to accurately estimate reasonable expected returns, which at least is their job to do.

Therefore, I would argue that today's capital markets are dominated by institutional investors that (1) are staffed by professionals who should be able to accurately determine expected (market) return, (2) can take an indefinite investment horizon, at least for the larger part of their budgets and (3) have large budgets that allow for spreading risk.

This enables them to develop strategies for investing in risky assets (like investing in market portfolios for an indefinite period) with an exposure that approaches the exposure of a riskless asset in the long-term, like an endless coin-tossing game. If you throw a coin a billion times, the deviation between the actual outcome and the expected return is negligible compared to the number of throws. So even if risk (uncertainty) does require additional return, which I do not see other than regarding liquidity and defaulting, institutional investors can largely mitigate this risk. Therefore, I believe institutional investors should behave more risk-neutral, which would reduce the equity premium on capital markets. And yet this is not the case…

The equity premium puzzle and the risk-free rate puzzle

So, if capital markets are dominated by institutional investors that have no reason to be risk-averse, why is it then that free capital markets do not self-correct and eliminate equity premiums? Even more so, research indicates that the risk-premium is too high and the risk-free rate is too low to be consistent with standard models[42]. This is called the equity premium puzzle and the risk-free rate puzzle in asset pricing theory. In the next two section I add the concept of "time value of time" that was introduced in chapter 4 into valuation and asset pricing such that this in my view better reflects the dynamics of valuation in the real economy. That way all existing mathematics remain valid, but the interpretation of both the equity premium and the risk-free rate is different. This view might add to solving both the risk-free rate and the equity premium puzzle. However, the answer requires market data analysis of a market that adopted the concept of individual time value of time.

5.2.2 Valuation dynamics in the real economy

The way I believe capital goods are traded in the real economy is perhaps best disclosed by an example. An old miller finds it increasingly difficult and painful to maintain its operations and produce flour, which results in reduced production hours, poor negotiation results with its buyers and suppliers and overdue maintenance of the mill. Therefore, his production volumes are decreasing, his margins are deteriorating, and his growth

[42] Back, a.o. page xx and page 170

perspectives are poor. His business becomes vulnerable because the mill increasingly malfunctions due to overdue maintenance investments. For this miller, the business case is poor. Even more so, as he is getting old his expected remaining lifetime is rapidly decreasing and volatility rises because of his fragile health condition. For this old man the wind mill has a low value. For Susy, who is a young ambitious woman on the other hand it is a great opportunity. She has plenty of energy to invest her time in proper maintenance of the windmill and to get better deals with customers and suppliers who also happen to be losing their edge as time passes. Because her parents were millers too, she sees all kinds of opportunities for growth investments like extending the blades to increase its power and output. For her, the windmill is valuable. If there is a monetary system in place that allows them to trade and to provide Susy with a mortgage backed loan to buy the windmill, they should be able to close a deal. There also was a competing bidder named John with the same age as Susy, but he was educated to become a baker so he didn't really have an opinion about the mill and failed to identify all its growth opportunities. Therefore, his bid was too low and Susy appeared to be the natural owner. This way, the old miller can enjoy a well-deserved retirement. He swapped the proceeds from selling the mill into life-insurance with a fixed income until he dies and he is comfortable knowing that his lifework is now in good hands that will continue to build what he once started when he was still young. And one day when Susy gets old, she will transfer her business to the natural owner that will take good care of her business and continue her work. This way people ensure that capital goods are owned by natural owners who are the ones that responsibly take care of and grow our heritage. We aim to improve it during our lives and then pass it on to the next generation. We try to make things a little bit better while we are on this planet anyway. If the incentives of the financial capital markets are aligned with real economy dynamics, I believe capitalism can facilitate natural ownership of capital goods and as such facilitate long-term sustainable growth.

5.2.3 Natural asset pricing and trading at capital markets

Let's see if we can propose a model that properly reflects these dynamics and then use it for natural asset pricing at the capital markets.

As just discussed, transactions of capital goods in the real economy occur because people have differing perspectives when valuing a capital good. The individual value of potential buyers and sellers is driven by (1) their plans with the capital good (i.e. the expected future growth of cashflows and the fraction of these cash flows that is required for investments in the capital good), (2) the predictability of these future cashflows and (3) the "time value of time" applicable to the potential buyer.

We can model these individual valuations with common corporate finance theories like the net present value method, except that this neglects the individuality of the "time value of time". Therefore, I believe we should interpret both the risk premium (or equity premium) and the risk-free rate differently as we do today which is further explained below.

Let's recall paragraph 4.2.4 "A life is like a stock price moving downwards". As an individual looks further into her future, she will realise that the chances on surviving are lower. For that reason, she will value present time over future time. Furthermore, young people that live a healthy life in a safe environment have more future perspective then old people and then people with a very adventurous lifestyle or in a dangerous environment. We proposed to consider the remaining expected lifetime of an individual *($T(t)$)* as Brownian motion[43], wherein the depreciation on the remaining expected lifetime is the drift. Both drift *(δ_T)* and volatility (σ_T^2) are driven by her lifestyle and environment and increase over time as she gets older.

For companies aiming for acquisitive growth, the dynamics do not seem to be very different. The drift now is the (annual) chance on default and comes with a certain volatility, both of which are related to (1) the sector dynamics wherein the company (potential buyer) operates and (2) its behaviour and associated risk-appetite (which for example is expressed by its financial structure and entrepreneurial ventures). The applicable risk-

[43] Brownian motion could be considered as a biased tossing-game or throwing weighted dice wherein the level of bias represents drift and determines the pace that biased outcomes occur more often than the other statistical outcome(s)

free rate would then reflect the "time value of time" of the potential buyer and could be modelled with Brownian motion with an interest component as drift that reflects the annual chance on default and a certain volatility (unpredictability).

With this "time value of time" in mind potential buyers observe companies (collections of capital goods) that they perceive as free cash flows (referred to as "assets") that are available at the capital markets. Based on its own knowledge and the sector dynamics of the target company the potential buyer projects an expected growth function (along with an expected required investment rate-function) on the present time free cash flow (at *t=0*) to estimate the expected future free cashflows. These free cashflows depend on the strategy the buyer has in mind with the asset. In analogy with the time value of time this growth function also could be considered Brownian motion, wherein the expected growth is the drift and comes with a certain volatility that represents the unpredictability of the expected growth due to the (lack of) buyer's expertise and the sector dynamics.

If we simplify both functions by assuming both drifts constant over time and add a discount respectively a premium to account for their volatility this results in a similar net present value formula as we know so well from corporate finance *(Value = FCF/(r-g))*. The difference is that the growth rate g' now is the expected growth rate g minus a discount μ_g to account for the individual unpredictability of the future cash flows. Secondly the cost of capital r' is the individual risk-free interest rate plus a premium μ_r to account for its volatility. Finally, the risk-free rate now reflects the time value of time of the buyer and does not relate to an alternative investment opportunity in an asset with a riskless return.

To see all this, we can recall formula (4.3) denoted in currency and apply it to a single company.

$$Y_{i+1} = \frac{(1+g(s))}{(1+\delta_0)} Y_i \Rightarrow \qquad FCF_{i+1} = \frac{(1+g')}{(1+r')} FCF_i\,,\ with\ r' = r_f + \mu_r\ and\ g' = g - \mu_g \qquad (5.2)$$

If we summarise this equation to infinity, we will obtain the value of the future expected cashflows which equals the classic net present value formula (5.3) that calculates the value of an eternal series of free cashflows with an annual constant growth rate of g and is discounted at a cost of capital level $r_f + \mu_r + \mu_g$.

$$Value = \frac{FCF_{t=1}}{r-g} = \frac{FCF_{t=1}}{(r_f+\mu_r) - (g - \mu_g)} = \frac{FCF_{t=1}}{(r_f+\mu_r+\mu_g)-g} \qquad (5.3)$$

Perhaps if we include this interpretation of the risk-free rate into the utility function of an individual aiming to maximise her utility (wealth) during the course of her life when developing factor models it could help us explaining the "equity premium puzzle" and the "risk-free rate puzzle". From what I understand matching real market data with factor models like the Capital Asset Pricing Model (CAPM) implies unrealistic risk-averse behaviour of investors at the capital markets. The equity premium at the real markets is higher than the factor models predict, and the risk-free rate is lower, both of which might be better explained by including the individual risk-free rate premium μ_r in the utility function of individual investors. That way, the depreciation rates of time r_f and a volatility μ_r of individuals and organisations that are in the market to buy assets are accounted for in pricing of equity premia and returns on risk-less bonds.

5.2.4 The Capital Goods Market Place

The role of debt is to enable transactions of capital goods (or assets) between people who want to be entrepreneurs (or companies that aim to grow) but do not have enough funds on the one hand and people who wish to have more leisure time (or companies with divestment strategies) and want to monetise their capital goods (assets). Young people or beginning entrepreneurs for example generally do not have enough funding to acquire the assets of retiring people, which would allow aged people to divest their business or home and retire or to live smaller and have more fixed-income funding for a pleasant retirement. Pension funds essentially swap the proceeds from selling the asset into a lifelong guaranteed income (like the miller from the example above did). Banks could base the interest rate they charge to entrepreneurs and companies on similar valuation perspectives to value the enterprise value (unlevered situation) of the company that requests financing and use Merton-Black-Scholes theory[44] to estimate the expected loss due to defaulting on the debt, which should be compensated by the interest rates they charge to their clients.

This I believe describes the natural dynamics of capital goods trading in the real economy. These dynamics are visualised in the matrix below (figure 5.3) that shows how different organisations take different views depending on the “time value of time” perspective and ability to forecast future cashflows. It explains for example that strategic buyers have an M&A advantage due to their specific sector knowledge (superior predictability) and presence in the sector of the acquisition target (synergies). Also, it might explain why institutional investors hold large stakes in fixed-income bonds. On these markets they do have an advantage due to their long-term view (low “time value of time”) and they do not have a disadvantage regarding the predictability of cashflows.

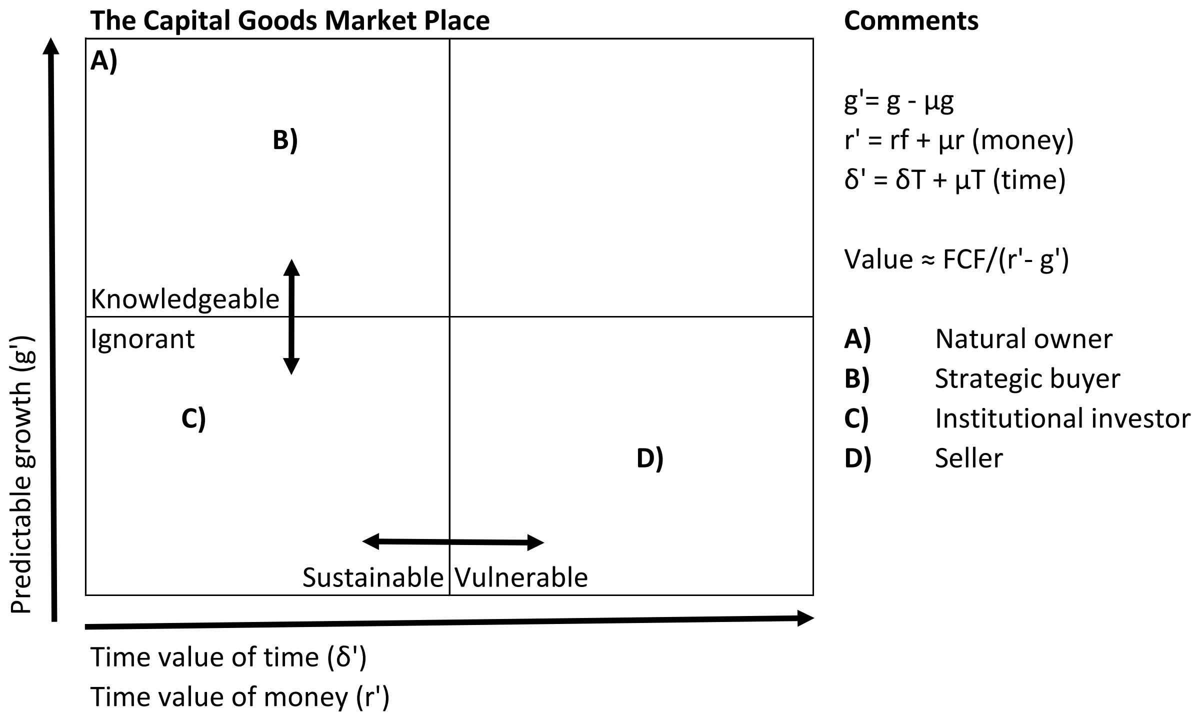

Figure 5.3 Capital Goods Market place

5.3 How aggregated time accounting relates to economics

5.3.1 Microeconomics and corporate finance

First let’s zoom in to see that every single transaction in the real economy is in fact an ownership transfer of useful time, which is accounted for in the financial system (ignoring the illegal and informal economy). To see this, please remember that in a real economy exist capital goods, consumption goods and services all of which can be denoted in and are exclusively the result of labour (useful human time). If one of these goods or services is transferred to somebody else the claim on the future expected benefits of the useful labour (either provided as a service or stored in the goods that are transferred) is also transferred to the new owner. In return the seller receives payment which is accounted for in the financial system. Transactions occur because the value of goods and services are individual and apparently, the willing buyer attributes more value to a good or service than a willing seller. Otherwise, there would be no agreement and hence no transfer of ownership. For services (like a software engineer or accountant) and consumption goods (like rice or gas) this is easy to see. Farmers that supply milk have more than they need for their own consumption (which is negligible related to their production volumes) so their minimal selling price is their best alternative opportunity to sell. The same holds

[44] https://www.mathworks.com/help/risk/default-probability-using-the-merton-model-for-structural-credit-risk.html

for the accountant or software engineer. And the buyers will base their maximum price on the minimum of (1) their expected marginally decreasing productivity increase or (2) their next best alternative opportunity to buy similar goods or services. We discussed valuation and transfer of capital goods separately in section 5.2. Other than that, all the dynamics like free markets, laws of demand- and supply, diminishing marginal returns and alike which we know from microeconomics do apply.

This way, any transaction within any economy is an exchange between time and money. This means that economic transactions yield time flows in the real economy that move in the exact opposite direction like cashflows do in the financial system (although not all labour or "useful time" is accounted for in money value). Companies have both consolidated time flows and cashflows and have the purpose to increase future labour productivity by developing and trading goods and services such that these goods and services are continuously used by the companies (and people) they are most useful to. The value companies create by doing so is partly reinvested in new investment opportunities that companies identify to further grow the business (i.e. increasing future labour efficiency). The remaining value that was created (but no longer considered useful, because the company does not see any more opportunities) is distributed because investors do not like excess cash (inactive money) in their companies and employees do not like to do useless things, nor do investors like this. This distribution in the financial system is the free cash flows that companies (sooner or later) return to their shareholders by means of dividend payments. In the real economy this distribution is the free time flows that are returned to the employees by means of dismissals or reduction of working hours. However, because (1) our economies on Earth are not well-functioning in the way we defined in chapter 4 and (2) we do not account for al lot of valuable activities (like job seeking) we cannot relate Free Time Flows $DIV_i = FTF_i = (1-\varphi)sY_0(1+g)^i$ to any existing financial metric. Nonetheless, I think it is fair to say on a micro-economic level that against the free cash flows of a company must be some form of a free time flow that remains unaccounted for, which must have its effect on the dynamics in the company. The reason is simple; in virtually all companies in capitalism-based economies labour and ownership are separated, hence the free time flows and free cash flows are distributed to different people. In other words, the people who have more time, because they managed to work and cooperate more efficiently do not receive the additional money (i.e. free cash flows) required to spend this time as additional leisure time (defined as increasing quality of life). Therefore, they do not have any incentive or opportunity to work less[45]. We will further analyse the impact of this phenomenon from an aggregated perspective in the next sections.

Summarising, companies create value for humans by continuously increasing labour efficiency. The free time flows and free cashflows this process releases are partly reinvested in the company for ongoing economic growth. The remaining money (excess cash) and time (redundant labour) is distributed back to respectively the shareholders (dividend payments) and the employees (dismissals).

This is how companies in well-functioning economies are supposed to create value for humans. I believe the concept works reasonably well in western economies regarding value creation. However, because (1) ownership and labour are largely separate in most capitalism-based economies such that people cannot spend additional free time as leisure time and (2) we do not account for a lot of valuable activities that people perform I would say this leaves a lot of room for improvement regarding distribution in capitalism-based economies.

5.3.2 Macroeconomics

The first thing we might want to understand when thinking of economic growth is that our measures like National Income and Gross Domestic product do not account for the free time flows that we discussed in chapter 4, which is the additional leisure time due to increasing labour productivity that people voluntarily spend at the cost of increased consumption. In formula this was time dividend $DIV_i = FTF_i = (1-\varphi)sY_0(1+g)^i$. The key difference is that the time spent by people that are involuntary unemployed in our economies on Earth is

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[45] This might (partly) explain the so-called bullshit jobs first described by David Greaber as jobs the people that occupy these jobs themselves think of as useless in his book Bullshit jobs

considered labour that is invested in the creation of capital goods when accounting and valuing in time. In economics we distinguish between the following three sorts of unemployment[46]:

1. Structural unemployment is unemployment due to obsolete skills of people;
2. Frictional unemployment is unemployment due to an everchanging demand and supply driven by economic growth such that sometimes people are in between jobs, but they do have economic value and hence will in time be employed again; and
3. Cyclical unemployment which is unemployment due to cyclical supply and demand dynamics such as economic cycles or seasonality.

It shows that in economics we consider all forms of unemployment involuntarily whereas in time accounting involuntarily unemployment should be accounted for as labour used to create capital goods and hence is capitalised on the aggregated balance sheet and depreciated over time. As far as I am aware, we do not register voluntary unemployment like leisure time of financially independent people, retirees and other people who can afford this. Because in the former chapter we saw that consumption growth is just half of the drivers of increasing quality of life, it might make sense to measure voluntary unemployment for the purpose of calculating the development of quality of life. And then still we would ignore the true values in life like happiness, relationships and peace.

So, from now on we should keep in mind that we consider economic growth as increasing consumption and do not account for increasing leisure time. When relating macroeconomics as we currently practice it to time accounting, we should therefore relate macroeconomic drivers to the "re-invested" aggregated time flows. For example, Gross Domestic Product should be somehow related to φY instead of Y, because $(1-\varphi)Y$ is unaccounted for. The difference might be not too material by the way, since as far as I am aware working hours of ordinary people (i.e. the vast majority of the people on the planet) have not been significantly decreasing since a hundred years ago[47].

Secondly, we need to be aware that looking for a job is not accounted for in real economies, although such activities do represent real economic value. Obviously, there are a lot of other valuable economic activities that remain unaccounted for in macroeconomics, like parenting and studying or travelling and building relationships during free time of employees. So, there seems no sensible way to mathematically relate macroeconomics to time accounting. Merely, we could follow an economy by practicing both time accounting and financial accounting and see where they diverge in order to better understand the underlying dynamics. For example, we could annually estimate the total labour hours spent and multiply this by the labour productivity growth[48] to estimate the real economic growth. If we adjust the aggregated production in a certain year the remaining growth must be inflationary. This inflation should then be equal to the calculated inflation (based on a basket of goods).

5.4 How macroeconomics relates to financial accounting

5.4.1 Gross Domestic Product and α

The main metric of macroeconomic growth is called Gross Domestic Product *(GDP)*[49], which for any given year *i* is defined as follows:

$$GDP_i = C_i + I_i + G_i + (X_i - M_i) \quad (5.4)$$

[46] Jason Clifford ACDC – Everything you need to know about Macroeconomics https://www.youtube.com/watch?v=MKO1icFVtDc
[47] Rutger Bregman states on page 33 and 34 of his book "Gratis Geld for Iedereen" (10e druk, December 2018) that John Maynard Keynes predicted in 1930 that we should have a 15 hour work week in 2030 and spending our ample leisure time would be our main challenge.
[48] In section 6.2 "Measurement of real economic growth and quality of life" we propose a method to calculate real economic growth
[49] Example given: https://en.wikipedia.org/wiki/Gross_domestic_product and https://www.bea.gov/system/files/2018-12/gdp3q18_3rd_1.pdf

In formula (5.4) *C* means the aggregate consumption of all people within the borders of the country, *I* means all investments made by companies in the country, *G* means all government spending, *X* is the value of all exported goods and services and *M* is the value of all imported goods and services. This formula is called the expenditure approach of the GDP. It is important to realise that GDP just represents what economists call the "real economy". There also exists a system of financial markets and capital markets where equity, debt and all kinds of derivative financial products and instruments can be traded (jointly referred to as "**Aggregated Financial Markets**"). In macroeconomic measures like GDP these markets are excluded because any and all profits and capital gains that were generated on these markets are purely monetary[50]. None of these profits and losses represent any underlying real economic growth driven by either (1) population growth or (2) labour productivity growth and therefore these profits are commonly neglected in deriving macroeconomically growth metrics like GDP.

Because every good or service that is sold is also bought by somebody else, there is an alternative approach to deriving GDP, which is called the income approach. In formula this is:

$$GDP_i = Rent_i + Interest_i + Profit_i + Wages_i \qquad (5.5)$$

Both approaches yield the same value of GDP by definition.

In a closed economy (like our global economy) there exist no export and import of goods and services. Therefore *(X-M)* equals zero in case of a closed economy. Also we should keep in mind that in a closed economy the aggregated interest and rent components in equation (5.5) must be zero (at least in our definition of *GDP* this must be so). This is because every interest and rent payment is also interest and rent received by someone else, like debt eliminates itself in equation (3.1). Unlike profit, interest and rent are purely financial metrics hence is a zero-sum game.

In macroeconomic literature rent, interest and profits are commonly jointly referred to as return on capital R_i to distinguish from income from labour L_i[51].

From now onwards we will refer to Gross Domestic Product as Y_i which is the total real production of all the people within a closed economy. This equals income received by return (R_i) on capital (K_i) and income from labour (L_i) from all economic activity in the real economy (and thus excludes all financial profits and gains generated at the aggregated financial markets). *GDP* is the equivalent (disregarding time dividends) of our definition of aggregated production in time accounting, hence we use the same symbol *Y*.

Here, capital (K_i) equals the sum of all registered ownership of capital goods in the real economy that generate registered cashflow (E_i) added with all Money (M_i), such that formula (3.1) $K_i = E_i + M_i$ still holds.

Using the *income approach* as described by formula (5.5) we can derive equation (5.6):

$$Y_i = GDP_i = Rent_i + Interest_i + Profit_i + Wages_i = R_i + L_i \qquad (5.6)$$

The fraction (percentage) of national income that is return on capital is commonly denoted by α. It is an important metric that shows how national income is divided by employees and owners and lenders (jointly referred to as Investors). From now on we will refer to this metric as $\alpha_{Piketty}$ (in order not to confuse the metric with our definition of α which will be defined as the consolidated gross margin of the private sector in the next section).

[50] At least this is what I conclude based on Wikipedia's description (https://en.wikipedia.org/wiki/Gross_domestic_product). If the financial sector is included in mainstream GDP definitions, my definition of GDP deviates from the formal one by excluding the financial sector, which does not materially impact the content of this book. We would need to adjust for this when plugging in real-life data though.

[51] Piketty's definitions of GDP, NGP, GNI and NNI (Piketty, pages 58 – 61) are equal to the definitions described in this book, except that unlike we Piketty seems to disregard Governments

$$\alpha_{Piketty,i} = \frac{R_i}{Y_i} \tag{5.7}$$

If we use formula (5.7) we can rewrite formula (5.6) into equation (5.8):

$$Y_i = R_i + L_i = \alpha_{Piketty,i} Y_i + (1 - \alpha_{Piketty,i}) Y_i \tag{5.8}$$

Piketty states that $\alpha_{Piketty}$ in western economies as a rule of thumb amounts to roughly 30% and depreciation typically is roughly 10%[52]. We should bear in mind though that Piketty uses Net Domestic Production (NDP, which equals Net National Income NNI in a closed economy) when measuring $\alpha_{Piketty}$. The difference with GDP is that in a closed economy we need to subtract depreciation and differences in production and consumption due to semi-finished products (i.e intermediate goods and services)[53] from the Gross Domestic Product. We will disregard differences in intermediate goods, but still need to bear in mind that Return on Capital R_i in Piketty's definition includes Depreciation (Dep_i). Because we consider R_i including depreciation based on our definition of $\alpha_{Piketty}$ (again as a rule of thumb) in the economies Piketty refers to must be roughly 30%*0,9 ≈ 27% (let's use 25%).

5.4.2 Consolidating the financial statements of a truly capitalistic closed economy

Now imagine a truly capitalistic closed economy. Truly capitalistic means that in the economy governments and banks do not own anything other than financial instruments like money, debt and loans. In other words, they just have monetary claims and positions and no have no financial instruments that represent a claim on any capital good in the real economy (referred to as Equity). Hence all capital goods in the real economy are ultimately owned by households. Closed means that there is no trading (or any other form of interaction) with other economies like our global economy. We assume there are no informal or illegal activities of companies, governments and banks. Finally, we assume that the government(s) in a truly capitalistic closed economy employ no people, but source all services from the private sector (i.e. companies). This assumption is largely done for the sake of analytical convenience, but nonetheless seems to fit in the concept of truly capitalism in a way that libertarians seem to promote a maximal private sector and minimal size of the government(s). If we would assume people employed by the government(s) this would introduce an additional monetary flow that we need to model in our equations and calculations, but it would not bring anything new to the table regarding the underlying dynamics and conclusions of this chapter. Taking all the assumptions into account, the following must hold:

1. All transactions between companies are properly accounted for in the financial statements of the companies involved (referred to as Business to Business transactions, or B2B);
2. All transactions of governments are properly accounted for in the financial statements of the companies involved (referred to as Business to Government transactions, or B2G);
3. All transactions between households and companies are properly accounted for in the financial statements of the companies involved (referred to as Business to Consumer transactions, or B2C);
4. All transactions between households like the sale of a house from one household to another are unaccounted for in any financial statement (referred to as consumer to consumer transactions or informal transactions, or C2C); and
5. All transactions between banks and households, companies or governments are properly accounted for in the financial statements of the bank involved (referred to as financial transactions)

All capital goods owned by companies are capitalised on the balance sheet (jointly referred to as aggregated equity E_i). Households also own capital goods, mostly houses, leisure boats, cars etcetera. However, because we defined this ownership as the informal economy all capital goods that were build and sold to households (like houses, cars and boats) are accounted for as consumption goods in GDP. All transaction between households (for example transfer of a second-hand car or a house) qualifies as the informal economy and is

[52] Piketty respectively pages 70 and 59
[53] Also referred to as "intermediate goods and services" https://en.wikipedia.org/wiki/Gross_domestic_product

disregarded. For simplicity we ignore indirect taxes and subsidies and will model unemployment benefits as if they were netted with labour taxes. Because governments in a truly capitalistic economy do not own capital goods or equity, they do not invest. Therefore, all government expenditures (G_i in formula 5.4) are consumption expenditures such that G_i equals our definition of the business to government market (B2G). Households might invest in their capital goods like painting their house, but in the financial statements of companies these goods are accounted for as consumption and not investments. Therefore, all expenditures of households in any given year in total equals our definition of the Business to Consumer market (B2C) and equals the definition of C_i in formula *(5.4)*. Therefore, in a truly capitalistic economy all investments that are accounted for are capital expenditures of companies and jointly equal the aggregated amount of investments I_i in formula (5.4).

Now imagine a large global merger (referred to as B2CG) of all companies operating in the real economy (i.e. this excludes financial institutions) in a closed truly capitalistic economy with the sole purpose to derive consolidated financial statements of all companies in the economy. If we disregard the Aggregated Financial Markets the total amount of aggregated sales of all companies in the real economy added together must equal Fisher's law[54]. This must hold true since all transactions that occurred in the real economy in any given year involves at least one company and hence is accounted for (we disregarded the informal economy). To derive the consolidated revenues of this merger (B2CG) we need to eliminate all intercompany transactions[55].

Because we defined B2CG as a merger of all companies in the economy, all business to business transactions (referred to as B2B) qualify as "intercompany transactions" that must be eliminated from the financial statements. Therefore, the aggregated consolidated revenues of B2CG amount to the remaining part of the formal economy which equals all transactions between companies on the one side and governments (referred to as B2G) or households (referred to as B2C) at the other side. In formula this is:

$$Consolidated\ revenues\ B2CG_i = C_i + G_i = Y_i - I_i = Y_i - sY_i = (1-s)Y_i \qquad (5.9)$$

In equation (5.9) we denoted the aggregated investments as a fraction *s* of the GDP and assume this fraction constant over time (equal to our approach in chapter 4).

If B2CG capitalises at historic costs, the aggregated consolidated amount of investments are solely labour costs (and includes no profit margin). Therefore, sY_i involves all labour costs in the economy related to the creation of capital goods owned by companies (E_i). By definition (i.e. by accounting rules) these costs are capitalised on the consolidated balance sheet of B2CG.

Now we define the consolidated EBITDA of B2CG relative to GDP(Y) as α_i, such that the EBITDA of B2CG in year *i* amounts to $\alpha_i Y_i$. Because all material costs are gradually transferred into either labour costs or profit margin during the process of consolidating, the EBITDA equals gross margin (or added value) in case of B2CG. In other words, all COGS and opex consist only of operational labour costs and depreciation, but no material costs. Rental costs are eliminated in the process of consolidation.

Piketty's alfa and the α we use are related as presented below, wherein Y_i' refers to the Net Domestic Product $(Y_i'=Y_i-\delta Y_i)$, τ_{CIT} is the consolidated corporate income tax as percentage of GDP and τ_{DIV} is the consolidated dividend tax as percentage of GDP, assuming a steady state (or observing long-term averages) economy with constant EBITDA margin (α), constant tax regimes and constant dividend pay-outs relative to the consolidated free cash flow and disregarding capital taxes.

$$\alpha_{Piketty}Y_i' = (\alpha - \tau_{CIT} - \tau_{DIV})Y_i$$

We can now use formula (5.9) to derive that the total annual labour costs spent on the production of consumption goods (and services) amounts to:

[54] https://en.wikipedia.org/wiki/Quantity_theory_of_money#Origins_and_development

[55] Also please refer to literature like "Accounting, text and cases" (Robert N. Anthony e.o., McGraw Hill, International Edition 2007) or search on internet for an explanation of the main principles (Wikipedia, Investopedia,…)

$$Consolidated\ operational\ labour\ costs\ B2CG_i = Consolidated\ Revenues\ B2CG_i - Consolidated\ EBITDA\ B2CG_i = (1-s)Y_i - \alpha Y_i = (1-\alpha-s)Y_i \quad (5.10)$$

In this equation, α is the consolidated EBITDA (which equals added value) of all companies in the economy relative to GDP *(Y)* netted from all taxes. Unless stated otherwise we will consider α constant over time ($\alpha_i=\alpha$ for all *i)* which implicates that wages are annually adjusted for inflation and real economic growth. This way we can derive the aggregated consolidated financial statements of B2CG and relate them to the components of GDP. We disregard (corporate) debt and interest for the moment, which makes no difference because we will merge B2CG with all financial institutions in the closed economy later such that all debt and interest costs are eliminated from the financial statements again.

CONSOLIDATED PROFIT AND LOSS ACCOUNT B2CG				
Financial accounting		**Macroeconomics**		**Comments**
Sum of all transactions of all subsidiaries of a group of companies	+	Sum of all recorded transactions in the economy	+	This must equal both sides of the Fisher equation[56]: $M*V_T=\Sigma (p_j * q_j) \approx P_T*T$
Eliminate intercompany transactions within groups of companies	-	Eliminate all business to business transactions in the economy (B2B)	-	Since B2CG is considered a merger of all companies in the economy all business to business transactions qualify as intercompany sales and should be eliminated to obtain the "consolidated" aggregated production value of the economy
Consolidated revenues	=	$C_i + G_i = Y_i - I_i = (1-s)Y_i$	=	If we eliminate all B2B transactions out of all economic activities, we obtain the added value of all companies operating in the real economy. This equals B2C and B2G and also equals GDP less aggregated corporate investments.
Operational expenditures (opex)	-	$(1-\alpha-s)Y_i$	-	Labour costs that were spent on production of consumption goods (i.e. accounted for as operational costs)
EBITDA *EBITDA as % of revenues*	=	αY_i α	=	Earnings before interest, tax, depreciation and amortisation
Depreciation *(Dep)* *Depreciation as % revenues*	-	δY_i δ	-	Amortisation ignored
EBIT *EBIT as % of revenues*	=	$(\alpha-\delta)Y_i$	=	Earnings before interest and tax
Interest (no interest)	-	-	-	Disregarded, because they will be eliminated in the merger with the financial sector shortly
Corporate income tax (CIT) *CIT as % of revenues*	-	$\tau_{CIT}Y_i$ τ_{CIT}	-	Corporate taxes
NOPLAT	=	$(\alpha-\delta-\tau_{CIT})Y_i$	=	Net operating profit less adjusted taxes equals net earnings of B2CG in case B2CG has no interest-bearing debt

Table 5.1 Profit and loss account of B2CG Inc expressed in both macroeconomics and financial accounting metrics

[56] https://en.wikipedia.org/wiki/Quantity_theory_of_money#Fisher's_equation_of_exchange. This ignores the presence of the Aggregated Financial Markets hence is true for the part of the total money supply that is active in the real economy.

CONSOLIDATED CASHFLOW STATEMENT B2CG INC				
Financial accounting		**Macroeconomics**		**Comments**
EBITDA *EBITDA as % of revenues*	=	αY_i α	=	See P&L for derivation
Interest (no interest)	-	-	-	Disregarded, because interest will be eliminated in the merger with financial sector shortly
Corporate income tax (CIT) *CIT as % of revenues*	-	$\tau_{CIT} Y_i$ τ_{CIT}	-	Corporate taxes
Working capital mutations *(0)*	-	Changes in intermediate goods	-	We disregard changes in stocks, and consolidated payables and receivables (all positions between B2CG and households and governments)
Capital expenditures (capex) *Capex as % of sales*	-	Investments (sY_i) s	-	Labour costs that is capitalised on B2CG balance sheet. Because we assume that investments are accounted for at historic costs (no margin is capitalised) these aggregated investments must equal (sY_i) and reflect the aggregated labour costs that were spent on investing in capital goods.
Free cash flow (FCF)	=	$(\alpha-\tau_{CIT}-s)Y_i$	=	Aggregated cashflows of all companies (except financial institutions) that is available for dividend payments. Or at least it is expected to remain inactive at the bank accounts of corporations to the extent not distributed back to its shareholders, because corporate finance theory dictates that this is excess cash for which companies do not see any more attractive investment opportunities.

Table 5.2 Cashflow statement of B2CG expressed in both macroeconomics and financial accounting metrics.

The consolidated financial statements show that the consolidated free cashflow amounts to $(\alpha-\tau_{CIT}-s)Y_i$. and the annual net consolidated profit amounts to $(\alpha-\delta-\tau_{CIT})Y_i$. In a mature steady state economy, s and δ must be roughly equal hence the fraction of GDP that is annually available for investors (i.e. the consolidated free cash flow) amounts to $(\alpha-\delta-\tau_{CIT})$. If we disregard the transfer of income by means of dividend taxes, the consolidated net profits (which roughly equals free cashflows in a mature economy) as percentage of the GDP $(\alpha-\delta-\tau_{CIT})$ is equal to what we defined earlier as $\alpha_{Piketty}$. This means that the consolidated free cash flow of all companies operating in the real economy of western economies roughly amounts to 25% of the GDP annually, disregarding the effect of dividend taxes and capital taxes which add up to just a small part of the tax income of governments[57]. Anyone with a background in corporate finance might intuitively get an uneasy feeling reading this. Corporate finance theory dictates that the free cash flows that companies create is money that is excessive, because companies see no more interesting growth opportunities to create more value for its

[57] According to Piketty $\alpha_{Piketty}$ amounts to roughly 30% of NDP (GDP-depreciation) in most western economies and estimates the annual depreciation is roughly 10%. Therefore we assume $\alpha_{Piketty}$ to amount to 25% of GDP an assume s equal to the depreciation rate (10%) which should be roughly true for mature economies on the long term.

shareholders. Therefore, free cashflows generally are distributed back to its shareholders as dividend payments.

The annual free cashflow $(\alpha-\tau_{CIT}-s)Y_i$ of B2CG is partly distributed to households as dividend payments $(DIV_i < FCF_i)$ and partly added to the balance sheet of companies as excess cash. The total Fixed Assets of B2CG's balance sheet therefore equals E_i, which represents an ownership claim on all capital goods in the economy. To come to the total amount of capital ("K_i") we should add the accumulated free cash flows that were distributed as dividends. The annual net contribution to the total equity (E_i) of B2CG equals the net investments (NI_i) which equals capitalised labour costs (sY_i) minus depreciation (δY_i). To calculate the increase in capital (K_i) we should add the net increase in cash to the net investments (NI_i), which equals Free Cash Flow minus dividend payments. The alternative route to calculate the annual net contribution to the total value of B2CG (ΔK_i) is to subtract dividend from NOPLAT. In formula we write:

$$DIV_i < FCF_i = (\alpha - \tau_{CIT} - s)Y_i \quad (5.11)$$

$$E_i - E_{i-1} = \Delta E_i = NI_i = (s - \delta)Y_i \quad (5.12)$$

$$Method\ 1{:}\ \Delta K_i = \Delta E_i + \Delta Cash_i = NI_i + FCF_i - DIV_i = (s - \delta)Y_i + (\alpha - \tau_{CIT} - s)Y_i - DIV_i$$

$$\Delta K_i = (\alpha - \tau_{CIT} - \delta)Y_i - DIV_i \quad \text{or}$$

$$Method\ 2{:}\ \Delta K_i = NOPLAT_i - DIV_i = (\alpha - \tau_{CIT} - \delta)Y_i - DIV_i \quad (5.13)$$

Now imagine another large global merger of all privately-owned banks and all other financial institutions in the economy. We refer to this merger as Finance to Consumers and Governments (F2CG). If we would merge F2CG with B2CG we would obtain the consolidated financial statements of the complete corporate economy and its balance sheet would represent the positions of all privately-owned businesses and financial institutions (netted from intercompany relations) against households, governments and central banks. We will refer to the merger of F2CG and B2CG as "**the private sector**". Similarly, we will jointly refer to all households and governments as the "**the public sector**". The consolidated financial statements of the merger of B2CG and F2CG (the private sector) do not differ much from the ones discussed above except that they now include all income from financial services. One might argue that there exists a grey area between financial services and services provided to the real economy. Insurance companies for example could be considered both as a financial services or real services. Also, big tech companies increasingly aim to provide financial services. However, since we now include both into one merger called the private sector it does no longer matter whether services qualify as real economy services (provided by B2CG) or financial services (provided by F2CG). We will assume that F2CG only provides (mortgage-backed and uncovered) loans and alike such as interest rate swaps such that we can denote its revenues as a weighted average retail interest rate *r* multiplied by the aggregated debt of all households and governments (rD_i). All other services provided to households and government are provided by B2CG. All services related to the Aggregated Financial Markets are eliminated in the process of consolidation, like all B2CG transactions. The consolidated financial statements of the private sector (the merger of B2CG and F2CG) are drafted below. Please refer to the spreadsheet model to see the financial statements of governments and households and how they relate to the financial statements of B2CG and F2CG.

CONSOLIDATED PROFIT AND LOSS ACCOUNT OF THE PRIVATE SECTOR (B2CG AND F2CG)				
Financial accounting		**Macroeconomics**		**Comments**
Consolidated revenues	=	$(1-s)Y_i + r_i D_{i-1}$	=	All revenues of B2CG and F2CG obtained from goods and services provided to governments and households. All intercompany transactions are eliminated hence the aggregated level of debt (*D*) is netted from all positions between B2CG and F2CG. Hence all company debt and debt positions between banks are eliminated.
Operational expenditures (opex)	-	$(1-\alpha-s)Y_i + (1-\alpha)r_i D_{i-1}$	-	In this example we assume α constant, which implicates that wages grow in line with inflation, real economic growth and growth of the financial sector. This is highly unlikely, in real life wage increases have been lagging inflationary and real economic growth for decades, not to mention adjustments for growth of the financial economy. The component $(1-\alpha)r_i D_{i-1}$ in fact reflects the labour costs of F2CG.
EBITDA *EBITDA as % of revenues*	=	$\alpha Y_i + \alpha r_i D_{i-1}$ α	=	Consolidated EBITDA of B2CG and F2CG
Depreciation *(Dep)* *Dep as % of B2CG revenues*	-	δY_i δ	-	We disregard any investments of F2CG hence implicitly assume that all F2CG's costs are labour costs
EBIT	=	$(\alpha-\delta)Y_i + \alpha r_i D_{i-1}$	=	
Interest (no interest)	-	0	-	All interest costs are eliminated
Corporate income taxes (CIT)[58] *CIT as % of revenues*	-	$\tau_{CIT} Y_i + \tau_{CIT} r_i D_{i-1}$ τ_{CIT}	-	B2CG and F2CG tax rate are equal based on %EBIT (see below)
NOPLAT	=	$(\alpha-\delta-\tau_{CIT}) Y_i + (\alpha-\tau_{CIT}) r_i D_{i-1}$	=	Net earnings of both B2CG and F2CG which equals Piketty's definition of R_i in his empirical research to study α disregarding dividend taxes, capital taxes and VAT

Table 5.3 Profit and loss account of the merger of B2CG and F2CG expressed in both macroeconomics and corporate finance metrics

Please note that the corporate income tax rate (τ_{CIT}) as a fraction of revenues for F2CG differs slightly from the rate of B2CG to account for the absence of depreciation such that they are equal when denoted as a fraction of EBIT ($\tau_{CIT,F2CG} = \tau_{CIT,B2CG} * (\alpha-\delta)/\alpha$). This adjustment is also made in the spreadsheet model.

[58] Due to the absence of depreciation in the financial sector τ_{CIT} should be a lower fraction of revenues in case of financial institutions. This is modelled properly in the supporting spreadsheet model but disregarded in the content of the book.

CONSOLIDATED CASHFLOW STATEMENT OF THE PRIVATE SECTOR (B2CG AND F2CG)				
Financial accounting		**Macroeconomics**		**Comments**
EBITDA *EBITDA as % of revenues*	=	$\alpha Y_i+\alpha r_i D_{i-1}$ α	=	See P&L for derivation
Interest (no interest)	-	*0*	-	All interest costs are eliminated
Corporate income taxes (CIT) *CIT as % of revenues*	-	$\tau_{CIT}Y_i+\tau_{CIT}r_iD_{i-1}$ τ_{CIT}	-	
Working capital mutations *(0)*	-	*0*	-	Changes in intermediate goods disregarded
Capital expenditures (capex) *Capex as % of B2CG revenues*	-	Investments (sY_i) s	-	Labour costs that is capitalised on B2CG balance sheet at historic costs.
Free cash flow (FCF)	=	$(\alpha-\tau_{CIT}-s)Y_i+(\alpha-\tau_{CIT})r_iD_{i-1}$	=	Consolidated free cash flows of B2CG ($(\alpha-s)Y_i$) and F2CG (αr_iD_{i-1}).

Table 5.4 Consolidated cashflow statement of the merger of B2CG and F2CG expressed in both macroeconomics and corporate finance metrics

The balance sheet of a bank looks slightly different, but essentially is the same as any other balance sheet. The assets of F2CG largely consist of loans outstanding (which in our merger with B2CG solely consists of loans provided to households and governments). Furthermore, the assets contain a certain cash position. On the liability side the company has an amount of savings, usually called deposits, which in our merger is all money owned by all governments and households and is kept at commercial bank accounts. Furthermore, F2CG has a certain position (usually a liability) against the central banks, jointly referred to as the central bank. The difference between all assets and liabilities as just discussed equals the equity value of F2CG.

Now imagine a third (and almost last) large global merger of all governments in the economy. Their consolidated financial statements are not too difficult. They do not invest and we disregard any working capital positions, hence the consolidated cashflow statement equals the consolidated profit and loss account. The consolidated revenues amount to all taxes raised from B2CG, F2CG, all labour taxes (which are assumed to include VAT and netted with unemployment benefits) and all dividend taxes (disregarding capital taxes). Costs involve all government consumption *(G)* and all interest payments on the consolidated amount of government debt. The difference is the net profit (loss) and equals the consolidated budget surplus (deficit) of all governments in year *i*. Since P&L and cashflow are equal the budget surplus (deficit) is also the consolidated free cash flow of all governments in the economy. The situation for households is quite similar. Consolidated revenues amount to all labour income (L_i) and dividend payments (DIV_i). Costs are all consumer spending (C_i) and interest payments on consumer debt. The difference between income and costs equals the consolidated free cash flow and amounts to the consolidated household budget surplus (or deficit).

Based on the financial statements above we can visualise the various consolidated monetary flows in a truly capitalistic economy between (1) the merger of B2C and F2CG, (2) households that do invest (entrepreneurs, retirees, well paid employees and alike), (3) households that fully rely on income from labour (or unemployment benefits) and (4) governments. In figure 5.4 below all symbols refer to the monetary flows as defined earlier. The symbol φ refers to the fraction of Net Domestic Product (Y') that households and governments jointly generate annually and use for consumption (C + G) next year. Hence (*1*-φ) is the savings rate (Θ) denoted as a fraction of Net Domestic Product *(Y')* that households and governments annually either keep at saving accounts or invest in financial instruments by trading at the Aggregated Financial Markets. This will be further discussed in the next paragraph.

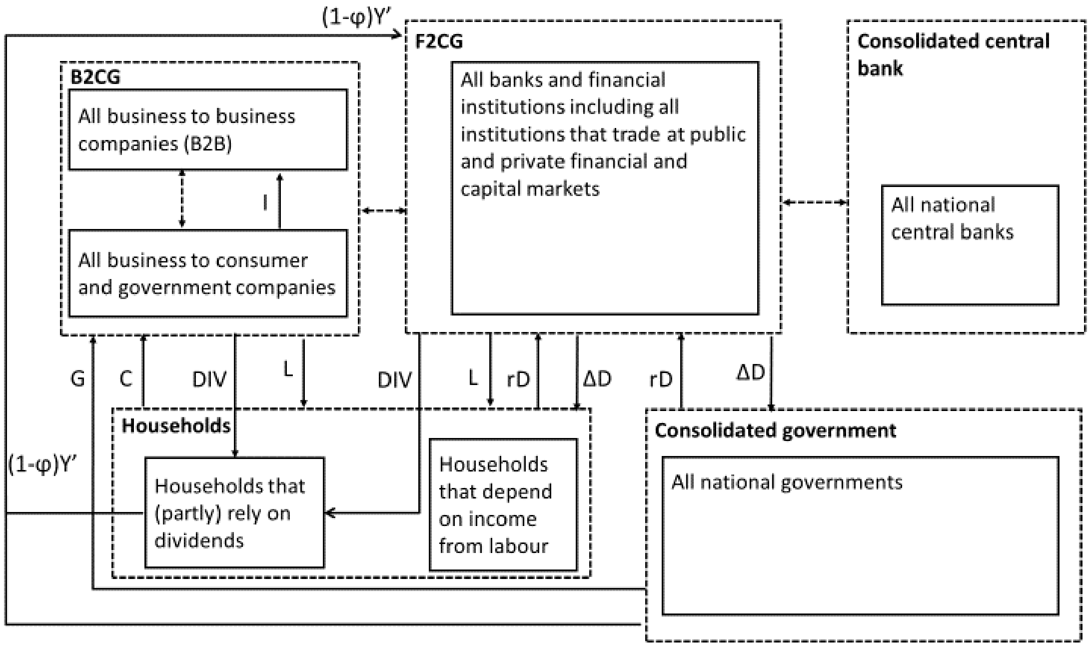

Figure 5.4 Visualisation of truly capitalistic closed economy with consolidated businesses, banks and governments. Dotted arrows represent interactions that are eliminated in the process of consolidation and can be disregarded. The role of the central banks regarding interest and debt creation will be discussed separately.

5.5 Financial instability

5.5.1 Global financial instability with risk-indifferent interest

In macroeconomics we distinguish between real growth (g_r) and nominal growth (g). The difference is inflationary growth (Δp) which is defined such that real growth over inflationary growth jointly are nominal growth:

$$(1 + g) = (1 + g_r) * (1 + \Delta p) \tag{5.14}$$

Since 1961 the global GDP annual nominal growth has never been negative, except for 2009[59]. I did not find any global statistics before 1961, but I assume that nominal growth between 1945 and 1961 also was positive on average. Hence it is fair to say that in general and on average nominal global GDP growth is positive (assuming absence of world wars). This is no surprise because most central banks aim for an annual inflation rate (Δp) just below 2%[60] and long-term average real annual growth of mature economies is also positive, roughly 1% to 1.5%[61]. Hence, if we use formula (5.14) it appears that the long-term nominal global GDP growth disregarding the additional growth of emerging economies should be roughly 3% annually. This is just something to keep in mind as a ball-park figure.

By definition the GDP's of any given year *i* and *i+1* are related to one another by formula (5.15), which states that the GDP of a certain year equals the GDP of the previous year added by real economic growth and inflation:

$$Y_{i+1} = (1 + g_i)Y_i = (1 + g_{r,i})(1 + \Delta p_i)Y_i \tag{5.15}$$

[59] https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG
[60] BoE: https://www.bankofengland.co.uk/monetary-policy/inflation, ECB: https://www.ecb.europa.eu/mopo/html/index.en.html, FED: https://en.wikipedia.org/wiki/Inflation_targeting
[61] Piketty, chapter 2 Groei: Illusie en Realiteit (pages 91-135, especially page 115.

If we take a long-term view on a truly capitalistic closed economy (i.e. disregarding short-term economic cycles of typically 7-10 years) equation (5.15) equals (5.16):

$$Y_{i+1} = (1+g)Y_i = (1+g_r)(1+\Delta p)Y_i \qquad (5.16)$$

From previous chapters we know that real economic growth g_r is driven by labour productivity increase *(ΔA)* and (labour-)population growth *(ΔL)* such that holds:

$$g_r = (1+\Delta A)(1+\Delta L)$$

Now, *g, g_r* and *Δp* are average metrics and assumed constant over time. This obviously is no longer an exact equation, but on the long-run (measured over many years such that short-term economic cycles can be disregarded) must hold true by definition. So, we need to bear in mind that we can only use equation (5.16) if we consider long periods of time, which is exactly what we intend to do.

Now recall that Net Domestic Product *(NDP=Y')* equals Gross Domestic Product *(GDP=Y)* less depreciation and note that (5.16) also holds for *NDP* if we assume a constant depreciation rate *(δ)*. If we use our definition of depreciation δY_i we can rearrange GDP and derive formula (5.17).

$$NDP_i = Y'_i = Y_i - \delta Y_i = C_i + G_i + I_i - \delta Y_i = C_i + G_i + (s-\delta)Y_i \approx C_i + G_i \qquad (5.17)$$

Formula (5.17) shows that if the depreciation rate roughly equals the investment rate of B2CG the net domestic product (Piketty's definition of national income) equals consumer spending *(C_i)* and government spending *(G_i)*. Because ultimately depreciation must equal investments, the long-term average investment rate *(s)* must equal the long-term average depreciation rate *(δ)*. Therefore, formula (5.17) must hold regarding the time windows we are interested in.

We will now define $\boldsymbol{\varphi_i}$ as the fraction of the aggregated consumer spending *(C_i)* and government spending *(G_i)* that will be reused for consumption by households and governments in the next year Y_{i+1}. Therefore, the savings in year *i* amount to (1- φ_i)($C_i + G_i$). This is money that is raised by governments and earned by households in any given year *i* and is not used for consumption the next year *i+1*. Therefore, the money is either kept inactive at bank accounts or used for buying securities at the aggregated financial markets (which would still keep the money inactive). In developed countries the saving rate seems to be typically between 5% and 10% of GDP[62], although in the US this is less. Our definition of φ_i will likely be (nearly) equal by a similar definition based on NDP which would intuitively make more sense. However, this definition yields mathematical simplicity. We will assume relation (5.17) holds and use the assumption that NDP_i equals ($C_i + G_i$) in any given year. However, if the assumption appears not accurate, we can always replace NDP_i by ($C_i + G_i$) and the maths would still hold. We will now assume that over long periods of time (1-φ_i) is structurally larger than *0* such that on average there is a net amount of money (1- φ_i)($C_i + G_i$) that annually drains away from the real economy because φ_i is structurally smaller than 1. The question is whether this is a reasonable assumption. Or is it fair to say that some years governments and households spend more than they receive and other years it is the other way around. I believe our assumption would make sense if there are at least some households and/or some governments that structurally have more income than they spend. Or we could go the other way around by assessing whether there are at least some households and/or some governments that structurally have more expenditures than income over long periods of time.

Do at least some households structurally save a fraction of their income?

An issue we need to tackle is that households are not static during the timelines we are looking at. Our time horizon is roughly 70 to 100 years at a minimum (at least long enough to ignore economic cycles)[63]. We could make complex bottom up calculations and follow households, descendants and inherited money generation by generation to see how wealth (or capital) spreads through populations by including inheritance and marital

[62]https://data.oecd.org/natincome/saving-rate.htm

[63] This seems to be the consensus timeline based on human experience and historic data that people consider as the ball park figure of the long-term economic cycle. Example given as observed by Ray Dalio in his book Principles of big debt crisis (www.principles.com)

wealth into the well-known utility-function[64]. Or we could use our common sense. We know that there exist wealthy families and individuals that still today can live from return on capital which was generated decades or even centuries ago. And we know that return on capital is structurally growing faster than both real and nominal economic growth[65]. This means that provided that the family wealth is large enough (an inheritance taxes are capital friendly) families could live from dividends and even increase their wealth for the next generation (we will get to this in the next paragraph). Furthermore, there are a lot of self-made entrepreneurs and investment bankers that structurally created more wealth and dividend proceeds than they will ever be able to spend during their lives. Finally, it is reasonable to assume that company dividends are generally lower than their free cash flows (or net earnings) hence households aren't even able to spend all the money they own, even if they would like to. Simply, because companies generally accumulate cash.

Next to wealthy families[66], pension funds and insurance companies own a lot of equity and fixed-income securities. The dividend and interest their portfolios generate are probably largely spend on retirement benefits and insurance coverage respectively. Furthermore, labour costs of people that are employed by the financial sector or suppliers to the financial sector like accountants, lawyers, fiscal advisors and alike all are partly paid from interest and dividends. These are monetary flows that are largely re-injected in the real economy.

Nonetheless, without doing the math we can see that obviously there are some households (or individuals) in the economy that structurally (i.e. over the length of their lifetimes) have less expenditures than their income. This implies that at least some households either (1) structurally save money or (2) invest part of their income on the Aggregated Financial Markets. Probably, they largely use this money for trading on the aggregated financial markets. They buy and sell securities, derivatives and equity (or they delegate this to wealth management firms). This annual increase in money available for trading at the aggregated financial market probably drives equity valuations and real estate prices in the real economy. As a result, average families and alike that own a house or some stock enjoy capital gain on their homes or assets which could stimulate them to spend more than their incomes from labour. For example by refinancing their mortgages, whereas (young) people that do not own a house have increasing difficulty to buy one[67].

Are there at least some governments that structurally spend more than they raise in taxes?

For governments the situation is no different, but because governments do not die and there are roughly 200 governments on the planet it is much easier to see if and which governments structurally spend more money than they receive. In fact, virtually all governments have a structural budget deficit. Norway and Qatar are two rare exceptions, which is easy if your country owns ample natural resources, just remember that citizenships are also equity. It is just a claim on future income. So in fact Norwegians and Qatar citizens are in a similar comfortable situation as second (and third and so on) generation wealthy family members. They inherited wealth that was earned or otherwise obtained by their ancestors.

Structural budget deficits that equal nominal economic growth (~3% per year) or less are generally considered prudent budgeting[68]. The rationale is that although the actual government debt annually rises, the amount of debt related to the nations GDP remains stable.

Having said all this, it seems reasonable to assume that there are at least some households that structurally save and at least some governments that structurally overspend

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[64] https://en.wikipedia.org/wiki/Utility
[65] Piketty's concept of r > g
[66] http://money.com/money/5054009/stock-ownership-10-percent-richest/
[67]See also Josh Ryan-Collins "Why can't you afford a home?" (Polity Press, 2019)
[68] https://en.wikipedia.org/wiki/Stability_and_Growth_Pact
https://en.wikipedia.org/wiki/United_States_federal_budget
https://www.bloomberg.com/news/articles/2018-11-12/china-expected-to-expand-budget-deficit-amid-trade-war-risks
https://www.ft.com/content/a9e5fb10-7087-11e8-92d3-6c13e5c92914

Summarising it is reasonable to assume that most governments and part of the households have a structural budget deficit. Also, at least some households have a structural budget surplus such that in any given year an amount of (1- φ_i)($C_i + G_i$)≈ (1- φ_i)(NDP_i)= (1- φ_i)(Y'_i) is withdrawn from the real economy and injected into the aggregated financial markets or kept inactive at saving accounts of households or companies. Therefore, φ_i on average is smaller than 1. We define φ as the long-term average fraction of the Net Domestic Product ($NDP_i \approx C_i + G_i$) in any given year that is used for consumption in the year thereafter by governments and households.

Now we can use figure 5.4 to see that the consolidated revenues of the private sector in year *i+1* must be funded by households and governments out of their incomes from year *i* added by a deficit (or surplus) which can only be funded by the private sector (merger of B2CG and F2CG). This must hold true since there is no direct line between the central banks and governments/household nor can governments/households create money. Therefore, the public sector has no other option than to accept loans from the private sector in case of budget deficits. Therefore, the following formula must hold true when assuming delayed payment of of taxes and labour by one period.

$$Consolidated\ revenues\ private\ sector_{i+1} = Income\ households\ and\ governments_i - government\ and\ household\ savings_{i+1} + \Delta D_{i+1}$$

In this formula ΔD_{i+1} is the total aggregated deficit (surplus) of all governments and households added together in year *i+1.* We can use the consolidated financial statements of the private sector (tables 4.3 and 4.4) and denote savings as a fraction *(1-φ)* of households and government income to express the formula above as formula (5.18)[69]:

$$Consolidated\ revenues_{i+1} = \varphi(Free\ Cash\ Flow_i + Operational\ labour\ icome_i + Capitalised\ labour\ income_i + corporate\ income\ taxes_i) + \Delta D_{i+1} \quad (5.18)$$

This can be written as follows.

$$(1-s)Y_{i+1} + rD_i = \varphi[(\alpha - \tau_{CIT} - s)Y_i + (\alpha - \tau_{CIT})rD_{i-1} + (1-\alpha-s)Y_i + (1-\alpha)rD_{i-1} + sY_i + \tau_{CIT}(Y_i + rD_i)] + \Delta D_{i+1} \quad (5.19)$$

The terms at the right side of the equation are from left to right (1) the FCF from the real economy (B2CG), (2) the FCF from the financial institutions (F2CG), (3) labour income from operational products and services provided in the real economy, (4) labour income from employees working for financial institutions, (5) capitalised labour income from employees that were working in the real economy, (6) corporate income taxes from the real economy (B2CG) and the financial services (F2CG and finally (7) the budget deficit that closes the gap.

Here, the average aggregated interest r_i is assumed constant over long periods of time at rate *r.*

If we simplify formula (4.19) we obtain formula (5.20).

$$(1-s)Y_{i+1} + rD_i = \varphi[(1-s)Y_i + rD_{i-1}] + \Delta D_{i+1} \quad (5.20)$$

On average) the nominal growth of the real economy equals *g*. We can use formula 5.16 to rewrite (5.20) into formula (5.21).

$$(1-s)(1+g)Y_i + rD_i = \varphi[(1-s)Y_i + rD_{i-1}] + \Delta D_{i+1} \quad (5.21)$$

Because $I_i = sY_i$ and assuming *s* constant over time we can rewrite $(1-s)Y_i$ into C_i+G_i, which roughly equals Net Domestic Product (NDP), because depreciation (δY_i) roughly equals corporate investments (sY_i) in the long run. This yields to formula (5.22)

$$(1+g)(C_i + G_i) + rD_i = \varphi[C_i + G_i + rD_{i-1}] + \Delta D_{i+1} \quad (5.22)$$

[69] In equations (5.18) and (5.19) the labour costs include (1) all labour income taxes, employers' contributions, VAT netted from unemployment benefits and other subsidies to households

Replacing $(C_i + G_i)$ by $NDP_i = Y'_i$, dividing both sides by Y'_i, and rearranging terms yields formula (5.23).

$$(1+g)Y'_i + rD_i = \varphi[Y'_i + rD_{i-1}] + \Delta D_{i+1}$$

$$(1+g) + r\frac{D_i}{Y'_i} = \varphi\left[1 + r\frac{D_{i-1}}{Y'_i}\right] + \frac{\Delta D_{i+1}}{Y'_i}$$

$$\frac{\Delta D_{i+1}}{Y'_i} = g + (1-\varphi) + \left[r\frac{D_i}{Y'_i} - \varphi r\frac{D_{i-1}}{Y'_i}\right] \quad (5.23)$$

In formula (5.23) we can easily see the various components of the annual deficit of the public sector (i.e. households and governments) relative to the National Domestic Product *(Y')*. These are (1) the nominal GDP growth *g,* (2) the annual savings of wealthy households and governments *(1-φ)* that draw money away from the real economy into the aggregated financial markets and hence leave a funding gap that apparently is compensated by spending of other households and governments in the real economy, (3) the ever-increasing amount of interest that must be paid to the private sector $(r(D_i/Y'))$ minus (4) the fraction of interest payments from last year that is reused for spending by the governments and households in the present year $(\varphi r(D_{i-1}/Y'))$. If we replace D_{i-1} by $D_i - \Delta D_i$, rearrange and reorganise the various terms we can obtain equation (5.24) out of (5.23):

$$\frac{\Delta D_{i+1}}{Y'_i} - r\varphi\frac{\Delta D_i}{Y'_i} - r(1-\varphi)\frac{D_i}{Y'_i} = g + (1-\varphi) \quad (5.24)$$

By replacing *(1-φ)* by the savings rate *Θ* an replacing $D_i - \Delta D_i$ with D_{i-1} we can express (5.24) as follows:

$$\frac{\Delta D_{i+1}}{Y'_i} = g + \theta + r(\frac{\Delta D_i}{Y'_i} + \theta\frac{D_{i-1}}{Y'_i}) \quad (5.25)$$

Let's refer to functions (5.23), (5.24) and (5.25) as the **exact public budget constraint**.

We can solve this numerically for any given g, r, D_0 and φ by using a spreadsheet (freely downloadable at buddhabanking.com). Don't ask me how I figured it out, but it appears that this function has a stable solution if $r(1-\varphi) < g(1-r\varphi)$. This implicates that this function remains stable within values which we would consider realistic. Although, we must realise that as φ moves away from 1 and *r* is larger than *g* the asymptotic level of debt relative to Y' (D_i/Y'_i) can be nasty. Example given, assume $\varphi=0.9$ (a saving rate of 10% of NDP plus interest income on public debt), *r=5%* and *g=3%* yields to an equilibrium debt to NDP level of nearly 5.5 and an annual interest *(rD)* of 27% of NDP. As a reference: our global debt level of both governments and households in 2018 was 1.5 times the global GDP[70], which roughly equals 1.65 times NDP. I was unable to solve the differential equation that is the equivalent of formula (5.24) in continuous time yet, but the solution seems to look like the following formula.

$$x(t) = \frac{g+(1-\varphi)}{g(1-r\varphi)-r(1-\varphi)}\left(1 - e^{(r(1-\varphi)-g(1-r\varphi))t}\right), with\ x(t) = D(t)/Y(t) \quad (5.26)$$

Which equals formula (5.27) if we denote (5.26) in terms of the savings rate *(Θ=1-φ).*

$$x(t) = \frac{g+\theta}{g(1-r)-r\theta(1-g)}\left(1 - e^{((1-g)r\theta-(1-r)g)t}\right), with\ x(t) = D(t)/Y(t) \quad (5.27)$$

We can simplify formula (5.23) by postulating that all income from debt (i.e $\varphi rD_i=0$) remains inactive in the financial system (i.e. is no longer reused for consumption) and adjust φ accordingly, which would relate φ to just the real economy (*Y'*). Therefore, the adjusted value of φ would then be larger than φ in formula (5.23) and could even exceed 1. We can derive a formula that relates both values of φ to one another using formula (5.23) as follows:

$$\frac{\Delta D_{i+1}}{Y'_i} = g + (1-\varphi_{new}) + r\frac{D_i}{Y'_i} = g + (1-\varphi_{old}) + \left[r\frac{D_i}{Y'_i} - \varphi_{old}r\frac{D_{i-1}}{Y'_i}\right] \Rightarrow$$

$$\varphi_{new} = \varphi_{old}(1 + r\frac{D_{i-1}}{Y'_i}) \quad (5.28)$$

[70]https://www.geotrendlines.nl/wereldwijde-schulden-naar-record-247-biljoen/

Implementing our postulation φrD_{i-1} equals 0 for any given year into equation (5.23) yields to the formula below.

$$\frac{\Delta D_{i+1}}{Y'_i} = g + (1-\varphi) + \left[r\frac{D_i}{Y'_i} - \varphi r\frac{D_{i-1}}{Y'_i}\right] = g + (1-\varphi) + \left[r\frac{D_i}{Y'_i} - 0\right]$$

We can be rearranged as follows:

$$\frac{\Delta D_{i+1}}{Y'_i} - r\frac{D_i}{Y'_i} = g + (1-\varphi) \tag{5.29}$$

We will refer to formula (5.29) as the **simplified public budget constraint**. We should then keep in mind that φ in formula (5.29) is φ_{new} in formula (5.28) and φ in formulas (5.23) – (5.25) is φ_{old} in formula (5.27).

Since $\varphi rD_{i-1}/Y_i$ is not constant over time at least one of the definitions of φ can also not be constant over time. We argued before that at least some households structurally consume less than they receive annually in the form of free cash flows (for example because not all free cash flows are distributed as dividend to households). Now, it is also reasonable to assume that the consumption level of these households grows with nominal economic growth which they can easily afford and is unrelated to their income, just because they structurally receive more than they want to consume. Their "spending elasticity" or "incremental spending rate" related to their income above the level of nominal economic growth is zero such that the consumption of these households is unsusceptible to their income. These dynamics off course must be tested with macro-economic data. If it appears largely true than all interest income *(rD_{i-1})* that are just monetary returns unrelated to growth and inflation in the real economy is abundant income. Since this ignores labour income from bankers (i.e all F2CG personnel) φ will likely not remain constant when interest income from public debt are significant relative to GDP. Hence, this effect is a stabilising factor at high public debt levels, although unfortunately it will draw away labour from the real economy into the financial sector. Nonetheless at reasonably low public interest costs we could assume that because generally speaking wealthy households have a consumption pattern that is largely unrelated to their income, income growth due to interest income is largely redundant. In that case φ_{new} does not exceed 1 and is likely more constant than φ_{old}. From now on we will not explicitly refer to φ_{new} or φ_{old}, because it evidently depends on whether we use the exact or simplified public budget constraint.

Although I am unable to proof it mathematically yet[71], it appears that the simplified public budget constraint (4.29) has a continues-time equivalent in the form of the following differential equation:

$$\frac{dx}{dt} - (r-g)x = g + (1-\varphi),\ with\ x(t) = \frac{D(t)}{Y(t)} \tag{5.30}$$

This differential equation has the following solution for boundary condition *x(t=0) = 0*:

$$x(t) = \frac{g+(1-\varphi)}{g-r}\left(1 - e^{(r-g)t}\right) \tag{5.31}$$

Please refer to the spreadsheet on buddhabanking.com to test this numerically. The spreadsheet models the evolution of debt over a period of 500 years for any given value of Y'_0, φ, *r* and *g.* Using formula (5.31) we can now easily see that for *x(t)* to be stable the weighted-average long term (commercial) interest rate on government debt and households debt *(r)* must be smaller than the nominal average long-term growth rate of the Net Domestic Product *(g)*, because *exp(r-g)t* must be negative to approach 0 for large *t*. In case *x(t)* is stable it has an asymptotic solution (g+1-φ)/(g-r) when *t* approaches infinity. We can also see that the level of re-consumption (φ) has no influence on the stability of the financial system, it only impacts the equilibrium debt level and the pace of convergence (or divergence in case of instability). Now, from macroeconomics we know that interest rates and inflation are closely related. If inflation goes up, the interest rates will likely go up too (and the other way around). Even more so, most people will feel that interest on debt should at least be equal to inflation. Otherwise, the loan would gradually loose its real value (adjusted for inflation). This is problematic though, because formula (5.31) tells us that as the interest *(r)* rate approaches nominal growth rate *(g)* the

[71] I think the way to go here is to replace D_{i+1} by $D_i(1+g)$ if this is correct. Than we could reduce the discretional time step i infinitely small such that ΔD_i can be written as *d/dt.* I am not yet sure this is a mathematically acceptable approach though.

equilibrium debt level will shift upwards and interest costs (*rD*) will rise. If for example the nominal growth equals 3% (i.e. roughly 2% inflation and 1% real economic growth) and the long-term interest rate equals 2%, the public debt-level will gradually move to 3 times *Y'* and the public interest costs *(rD)* will be 6% of *Y'*. However, if the interest rate equals 2.5% the debt level will grow to 5.5 times *Y'* and interest costs amount to nearly 14% of *Y'*. if retail (B2C and B2G) interest rates are structurally higher than nominal growth *(r > g)* the debt-level will diverge (accelerate into an infinite debt level) in the long-run. This implies that inevitably sooner or later households and/or governments will default if public interest rates *r* are structurally exceeding nominal NDP growth *(g)*.

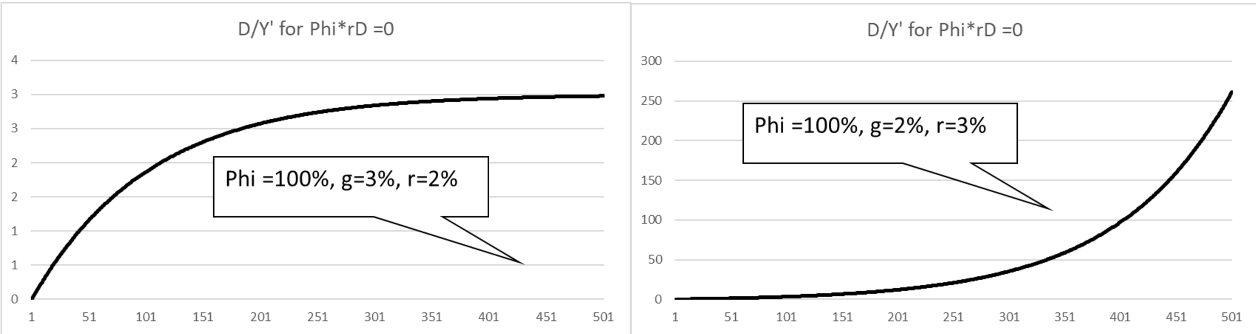

Figure 5.5: Equation (5.29) plotted for a period of 500 years with φ=100%, r=2%, g=3% (left side) and with φ=100%, r=3%, g=2% (right side) respectively.

5.5.2 Global financial instability with risk aversity

So far, we assumed that the interest rate *r* that banks charge to governments and households is indifferent from the leverage (amount of debt relative to their income) of the households and governments. This obviously is not the case. In real-life the interest rate rises with increasing leverage, because the chances on default rise with increasing leverage. Hence, we can say two things:

1. The interest rate *r* is a function of the amount of public debt relative to the NDP; *r=f(x)*, with *x=D/Y'*; and
2. The function *r(x)* is convex. This means that $r(x_2)$ is larger than $r(x_1)$ if x_2 is larger than x_2

Without knowing exactly how the interest rate relates to *D/Y'*, we can still conclude that because r(x) is convex this adds to the instability of the (simplified) public budget constraint. In other words, because interest rates will rise with rising public debt relative to NDP, the financial system can be instable even when the public budget constraint predicts stability. Including r(x) into the continuous time simplified public budget constraint yields formula (5.32).

$$\frac{dx}{dt} - (r(x) - g)x = g + (1 - \varphi),\ with\ x(t) = \frac{D(t)}{Y(t)} \quad (5.32)$$

We could for example assume r(x) to be linear $r(x)=r_0(x)$ or exponential $r(x)=exp(r_0x)$ and solve formula (5.32). Unfortunately, my mathematical skills are currently insufficient.

5.5.3 Is our global financial system currently stable?

Please find below a rough check based on publicly available data of global governmental and household debt[72]. Furthermore, I used US inflation[73] (which might not be an acceptable proxy for global inflation) and had to obtain the 2003 global GDP from a different source than the debt data[74].

Global debt in USD Trillion (% of total)	**2003**	**2018**	**CAGR 2003-2018 (%)**
Financial institutions	29 (29%)	61 (25%)	5.20%
Companies	29 (29%)	70 (28%)	6.16%
Governments	23 (23%)	67 (27%)	7.35%
Households	20 (20%)	49 (20%)	6.21%
Total global debt	**100 (100%)**	**247 (100%**	**6.21%**
Global GDP (USD Trillion)	37	78	5.06%
Average (US) annual inflation			2.09%
Implied average annual global real growth			2.91%
Global debt divided by GDP (Y/D)			
Financial institutions	0.77	0.79	0.13%
Companies	0.77	0.90	1.05%
Governments	0.62	0.86	2.18%
Households	0.54	0.64	1.10%
Total global debt (x GDP)	**2.70**	**3.18**	**1.10%**

Table 5.5 Global debt levels and its growth rates between 2003 and 2018.

We can use the data from table 5.5 and insert this into equation (5.31) to estimate *r* if we set φ at 1 (all interest income is saved and all other earnings are re-consumed). This results in a weighted average interest rate of 3%. This is smaller than 5% annual GDP growth and hence suggests global financial stability at an equilibrium debt level of 5%/(5%-3%) equals 2.5 times GDP. If this is true and growth, interest rates and national fiscal regimes remain largely unchanged we can expect that the consolidated debt level of the global public sector will gradually grow from 1.5 to 2.5 times global GDP in the next decades. The public sector would then annually pay to the private sector an amount of 2.5 times 3% which equals 7.5% of GDP of interest *(rD)* to the private sector. Instability or disruptions could be triggered by:

- Central banks increasing the (historically low) interest rate levels;
- A slowdown of the global real economic growth (which eventually is more than likely to happen[75]);
- A slowdown of the global inflation rate; or
- A decrease in consumption patterns of wealthy households such that φ decreases (although this only raises the equilibrium debt level and would not trigger instability).

We could also use these variables in our spreadsheet “5.6 Financial Equality” that allows for more sophisticated modelling. It models tax flows for example. If we assume equal parameters as in the exercise above (table 4.5) along with 26% corporate income taxes (as % of EBIT) a 100% pay-out of Free cashflows, 15% dividend taxes (as % of dividends) and a 3% interest rate the model calculates a 95% re-consumption rate between 2003 and 2018 to match the debt levels of 2003 and 2018. This implies a savings rate of 5% of GDP. If the parameters remain unchanged the public debt would gradually grow to 1.85 times GDP in the next 50 years and public interest would amount 5.3% of GDP. Other things equal, if saving rates would grow to 10% (which is typical for north

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[72] https://www.geotrendlines.nl/wereldwijde-schulden-naar-record-247-biljoen/
[73] http://www.in2013dollars.com/2003-dollars-in-2018?amount=100
[74] https://data.worldbank.org/indicator/NY.GDP.MKTP.CD
[75] Piketty concludes that mature economies (like Western European economies and the USA) have never managed to grow by more than 1.5% annually over long periods of time

western European countries like Scandinavia and the Netherlands) then the global public debt would grow to 3.2 times GDP with an interest burden of 9.1% of GDP.

These numerical exercises are just quick sanity checks with inaccurate data. Obviously, the time frame is way too short and the data are indicative at best. It is meant to show how we could verify (or reject) formula (5.31) and the public budget constraints from real data and use it to predict future debt- and interest levels between the private sector and the public sector.

5.5.4 Financial instability of a single government

Now let's have a look at an individual government to see how governmental debt evolves.

Assume a government of a closed economy with a fixed fiscal taxing regime in a steady state economy such that α, *g* and *r* remain constant over time. The government annually spends G_i and annually raises T_i every year in taxes. The government has a structural budget deficit such that T_i is φG_i at any given year *i*, with constant φ. Therefore, the annual budget deficit equals $G_i - T_i$, which equals $(1\text{-}\varphi)G_i$. Interest costs are paid at the end of every year based on the amount of debt at the beginning of the year (ΔD_i). Also, taxes are paid at the end of every year. The government maintains a policy to increase is spending in line with nominal growth of the economy *(g)*. The following equation regarding the financing constraints of the government then holds:

$$G_{i+1} + rD_i = (1+g)G_i + rD_i = T_i + \Delta D_{i+1} = \varphi G_i + \ \Delta D_{i+1}$$

In this equation *r* refers to the interest rate on government debt. In words this equations means that government spending and interest costs in year *i+1* are paid with all government tax income from year *i* added with an amount that the government needs to borrow to fill the budget deficit. This can be rewritten as formula (5.34).

$$\frac{\Delta D_{i+1}}{G_i} - \ r\frac{D_i}{G_i} = g + (1-\varphi) \tag{5.34}$$

This equation equals the simplified public budget constraint equation (5.29) and has the following solution in continuous time for $x=(D_G(t)/G(t))$ and $x(0)=0$:

$$x(t) = \frac{D(t)}{G(t)} = \frac{g+(1-\varphi)}{g-r}\left(1 - e^{(r-g)t}\right) \tag{5.35}$$

This equation shows that once interest on government debt increases above the inflation rate *(r > g)* governments are forced to adjust their budgeting practices in order to prevent defaulting (i.e. reduce the annual increase of the government to a level of growth that is smaller than the interest rate *r* the government has to pay on its debt.)[76]. We should keep in mind that we assumed a risk-neutral interest rate such that *r* remains constant despite increasing debt relative to government income. This might seem unrealistic at first glance, because in real life interest rates will increase with increasing leverage. Probably the right way to model this is to account for the chances on default, such that the incremental increase of interest matches the incremental chance on default multiplied by the expected loss for the lenders in case of default. The theory that does exactly this is first developed by Robert Merton in 1974[77], which is closely related to the Black-Scholes option pricing theory. However, for the sake of stability central banks will do whatever is in their power to maintain a stable monetary system. To do so they have no choice but to ensure that the governmental interest rates remain below nominal GDP growth. No surprise that stimulating inflation and reducing interest rates is exactly what the central banks of the largest mature economies (i.e. USA, European Union, United Kingdom and Japan) have been doing for at least the past decade.

Arguably Japan is the best example of a nation that enjoys artificially low interest costs. Although the governmental debt of Japan amounts to more than 8 times the level of 1985 its nominal interest costs are

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[76] This is a well known and widely accepted conclusion, for example see Elmendorf and Mankiv 1998 (https://scholar.harvard.edu/files/mankiw/files/govdebt.pdf)
[77] https://www.fields.utoronto.ca/programs/scientific/09-10/finance/courses/hurdnotes2.pdf

lower[78]. In 2017 Japan paid negative interest on a 5 year bond and below 1% interest on a 20 year bond. Imagine what would happen if the interest rates on Japan's debt level of more than 2 times its GDP[79] will go back to historic average rates. This will probably not happen in the foreseeable future because nobody would like this to happen. Nor can anyone oversee the consequences of such an event. In the meantime (like forever?) the Bank of Japan will do anything within their power to ensure that the interest rate that the government pays will stay below the level of inflation to keep the situation manageable.

It appears that it might not be worthwhile the effort to include a risk-averse interest *r(x)* based on Robert Merton's default probability theories in the public debt constraint like we suggested in section 4.5.2. This would only make sense if the laws of efficient free markets would apply on public financial markets. Obviously, they do not. In the aftermath of the financial crisis of 2008 we learned that we apparently have system banks that are just too big to fail. It appears to me that we also have system governments. Because central banks are responsible for maintaining financial stability, they will do whatever it takes to avoid defaulting system governments.

5.5.5 Are Ponzi schemes inevitable in a fractional reserve banking system?

From the simplified public budget constraint equation (4.29) we can see that for every given year *i* the increase in public debt is driven by (1) nominal NDP growth (g_i), (2) the interest payments on existing debt (rD_{i-1}) and (3) the saving rate $(1-\varphi)$ of income from the real economy that is saved instead of used for consumption.

$$\frac{\Delta D_{i+1}}{Y'_i} = g + (1-\varphi) + r\frac{D_i}{Y'_i} \qquad (5.29)$$

Now assume *D* equals zero in year 0 and assume households and governments do not save. Even then the public sector requires and amount of gY_0 in year 1 to cover for the nominal growth (that generally is larger than 0%). In year 2 the public sector requires an amount gY_1 to cover its budget and requires an amount of rgY_0 to pay for its interest obligations out of their debt. This will happen again in year 3 and beyond. So even if all households and governments have no debt and spend all their income from the real economy from last year such that the aggregated saving rate (1-φ) is zero they still must borrow year on year from the private sector to cover the nominal growth.

Therefore, provided that the simplified budget constraint is a fair representation of the public budget constraint, in a truly capitalistic closed economy with a fractional banking system that has a certain NDP with an average long-term nominal growth rate of *g* larger than 0% it is inevitable that the public sector borrows from the private sector to fund the growth of the economy (NDP). This debt position triggers interest payments from the public sector to the private sector. Because the public sector needs to borrow to fund next year's growth, it also needs to borrow the money to pay the interest on existing debt. Funding interest obligations (rD_{i-1}) by issuing new debt is called a "Ponzi-scheme"[80] and is generally considered unsustainable or even fraudulent[81] in finance and economics. However, in a capitalistic economy with a private sector that has positive free cashflows and a fractional reserve banking system at least some households and governments have no choice but to fund interest with raising new debt. This is because of the simple reason that companies and banks generally have positive free cash flows whereas governments and at least some households generally have not. In other words, we cannot all make money. Finance is a zero-sum game, so someone's gain is somebody else's loss.

It would be more accurate if we would use the exact public budget constraint (5.23) instead of the simplified constraint (5.29) to look at this problem.

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[78] https://www.bloomberg.com/news/articles/2018-05-13/kuroda-s-stimulus-saves-japan-45-billion-easing-debt-pressures
[79] https://www.ceicdata.com/en/indicator/japan/government-debt--of-nominal-gdp
[80] https://scholar.harvard.edu/files/mankiw/files/govdebt.pdf at the bottom of page 47
[81] https://en.wikipedia.org/wiki/Ponzi_scheme

$$\frac{\Delta D_{i+1}}{Y'_i} = g + (1-\varphi) + \left[r\frac{D_i}{Y'_i} - \varphi r\frac{D_{i-1}}{Y'_i}\right] \tag{5.23}$$

Again we could start in year *0* with zero public debt and assume that all households and all governments spend all their income (φ=1). Again the public sector would spend gY'_0 more in year 1 compared to year 0, which they could not fund. Therefore, the debt at the beginning of year 2 would also be gY'_0. The difference with the simple public budget constraint situation is that all interest income from the prior year is consumed again next year. This would significantly reduce the pace of debt increase, because the public sector only requires issuing debt to fund the nominal NDP growth and to cover for the interest cost of just one year. So still a fraction of new debt to be raised annually by the public sector is meant to cover interest obligations, although it is just one year of interest payments. Nonetheless, technically this still qualifies as a Ponzi scheme although you might also consider it a bridgeloan provided to the public sector. However, in case of the exact public budget constraint it is irrational to assume a saving rate ($1-\varphi_{old}$) of 0. It would implicate that (1) all companies distribute 100% of the free cash flows at the end of each year to their ultimate shareholding households and (2) all these (and other) households would spend 100% of their income every year on consumption in the real economy. We know that both assumptions are unrealistic. Companies do not distribute all their free cash flows and rich families save cash and use it for investing in financial instruments like bonds and stocks. Typically, the saving rate of western countries varies between 5% and 10%[82]. Therefore, also from this point of view we know that in a capitalistic economy with a profitable private sector and a fractional reserve banking system it is inevitable that at least some households and governments structurally borrow to fund interest obligations (commonly referred to as a Ponzi scheme). This "Ponzi-component" equals the last term $(r\Theta D/Y')$ of the exact public budget constraint (5.25).

$$\frac{\Delta D_{i+1}}{Y'_i} = g + \Theta + r\left(\frac{\Delta D_i}{Y'_i} + \Theta\frac{D_{i-1}}{Y'_i}\right) \tag{5.25}$$

The third term $(r\Delta D/Y')$ could be considered a "bridge loan" to bridge the time lag between tax income and/or labour income and their spending.

Based on our quick analysis of our current global situation ($g=5\%$, $\Theta=5\%$ and $r=3\%$) formula (5.25) predicts that roughly 2.2% of the annual debt increase (i.e. $((r\Theta D)/Y/(g+\Theta+r\Theta D/Y)$ qualifies as a Ponzi-scheme.

Probably, it is largely governments that borrow to cover for interest payments over longer periods of time. If households would do this, they would sooner or later default on their debt. To put it differently, the period that households can survive budget deficits is considerably shorter than in the case of governments. An example of a government with both a significant budget deficit and significant interest costs is the USA (USD 1.101 trillion budget deficit and USD 479 billion of interest costs)[83].

Obviously, governments have the option to increase taxes to solve any Ponzi-schemes they are caught in. We should bear in mind though that finance is a zero-sum game. If they raise labour taxes they shift their problem to households depending on labour income. If they raise income taxes corporate profits and cashflows $(\alpha-s)Y_i+\alpha r_i D_{i-1}$ would go down. However, even if governments would raise taxes on profits and dividend such that it would draw away all free cash flows from the private sector (such that $(\alpha-s)Y_i+\alpha r_i D_{i-1}\approx 0$) it would still leave a gap of *g* percent, besides the fact that implying a nearly 100% corporate tax rate is totally unrealistic. To me, this raises the question if fractional reserve banking (or any other money creation system that obliges the public sector to borrow from the private sector to fund nominal NDP growth) is a sustainable financial system. To put it differently, I wouldn't know how the public sector is supposed to close its budget deficit. Hence, there will always be countries who have no option but to default or to inflate their debt away every now and then.

Private sector debt

Finally let me say that we largely disregarded debt from the private sector (company debt and debt positions between commercial banks), because we eliminated this in consolidating the financial statements. Although it appears from table 5.5 that private sector debt grows below average, it still would be worthwhile to

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[82] https://data.oecd.org/natincome/saving-rate.htm
[83] https://www.thebalance.com/u-s-federal-budget-breakdown-3305789

understand why the debt level of corporations seems to outgrow nominal economic growth. This is surprising to me, because the private sector (generally speaking) can easily finance its growth from their excess (free) cash flows. From table 5.4 we know that the consolidated free cash flows of the private sector equals $(\alpha-\tau_{CIT}-s)Y_i+(\alpha-\tau_{CIT})r_iD_{i-1}$. This splits into $(\alpha-\tau_{CIT}-s)Y_i$ for all companies in the real economy (B2CG). Since $\alpha_{Piketty}$ (netted from taxes) is roughly 25% in western economies and s roughly 10% (in line with depreciation) companies and should have ample free cash flow (15% of GDP) to fund the annual growth g (<5% of GDP). Although off course within the private sector there are all sorts of companies with different financing needs and growth perspectives. This could mean that companies are largely contracting debt to leverage their equity returns. Banks grow their businesses by issuing loans to companies, governments and households. We'll get to the way banks create money later, which is commonly done by fractional reserve banking. For now, let me say that generally speaking banks must keep roughly 10% of all inflowing money (either their own profits or customer deposits). This means that their annual free cash flow of $(\alpha-\tau_{CIT})r_iD_{i-1}$ easily enables the financial sector to issue new debt to fund growth requirements of households and governments (likewise this holds for funding the growth of companies). This is because $(\alpha-\tau_{CIT})r_iD_{i-1}$ enables banks to issue roughly 10 times αr_iD_{i-1} of issuance of new loans, where the multiplier of 10 is called the money multiplier which is one over the fraction that banks are obliged to keep by the central bank's regime they are subject to. Hence, annually banks can roughly issue 10 times $(\alpha-\tau_{CIT})r_iD_{i-1}$ which equals 10*25%*3%*1.5*Y' = 11% of Y' or 10% of Y. Households and governments require lending gY' (3% to 5% of Y') for nominal NDP growth, hence the rest is available for funding interest costs. However, we disregarded the annual savings of households and the private sector (undistributed free cash flows). Summarising it should be no surprise that governmental debt (and household debt) is growing at a faster pace than debt of the private sector.

5.5.6 The net public budget constraint and the net interest rate

As discussed before the exact public budget constraint seems to be more stable than the simplified public budget constraint with a constant φ. The reason for this is that in fact a significant part of the public interest payments are recaptured by governments and households by means of taxes, dividend payments and labour income related to the financial sector. Therefore, interest payments are largely reused by the public sector which limits the increase of debt hence keeps the financial system stable. The component $\varphi r(D_{i-1}/Y_i)$ in the exact budget constraint (5.23) represents this fraction of interest payments that is reinjected in the real economy.

$$\frac{\Delta D_{i+1}}{Y'_i} = g + (1-\varphi) + \left[r\frac{D_i}{Y'_i} - \varphi r\frac{D_{i-1}}{Y'_i}\right] \quad (5.23)$$

So, if we redefine the interest r such that this just resembles the interest that is not regained by the public sector by means of labour, dividend and taxes, we can rearrange the exact public budget constraint into an equivalent form of the simplified public budget constraint without losing any accuracy (assuming constant taxes, dividend pay-out ratios, profit margins and nominal growth). So, let's define the net interest rate r_{net} as presented by formula (5.36).

$$r_{net}\left(\frac{D_i}{Y'_i}\right) = \left[r\frac{D_i}{Y'_i} - \varphi r\frac{D_{i-1}}{Y'_i}\right] \quad (5.36)$$

Implementing this into the exact public budget constraint formula (5.23) we have redefined the exact public budget constraint in the form of the simplified public budget constraint like formula (5.37):

$$\frac{\Delta D_{i+1}}{Y'_i} = g + (1-\varphi) + r_{net}\frac{D_i}{Y'_i} = g + \theta + r_{net}\frac{D_i}{Y'_i} \quad (5.37)$$

Let's refer to equation (5.37) as the net public budget constraint. The solution of the continuous time differential equation which is the equivalent of the net public budget constraint has a solution with x(t=0)=0 and x(t)=D(t)/Y'(t) like formula (5.38).

$$x(t) = \frac{D(t)}{Y'(t)} = \frac{g+\theta}{g-r_{net}}\left(1-e^{(r_{net}-g)t}\right) \quad (5.38)$$

All we need to do now is to express the net interest rate r_{net} in terms of tax rates, dividend pay-out ratio, growth and profit margin applicable to the financial institutions. To do so, we need to understand that the annual amount of public interest that is not reinjected into the real economy *($r_{net}D$)* is just the part of the annual free cash flow of the consolidated financial sector (F2CG) that is not distributed as dividend payments. This also is exactly what equation (5.36) tells us. The net interest over the public debt in year *i* equals all income *(rD_i)* minus all parts of last year's income that is redistributed to the public sector *($\varphi rD_{i-1}/Y'_i$)* and hence available for reusage by the public sector.

Having a look at table 5.4 can help in understanding this, for example by expressing the annual increase in excess cash of the financial sector (F2CG) by using the consolidated cashflow statement of the financial sector. The free cash flow of F2CG in year *i* amounts to $\alpha_{Piketty} r_i D_{i-1}$ (remember we disregarded investments in capital goods of the financial sector). In this equation $\alpha_{Piketty}$ is netted from all taxes. So, the free cash flow is $(\alpha-\tau_{CIT}) r_i D_{i-1}$, wherein α is the added value (or EBITDA) margin and τ_{CIT} is the corporate income tax rate expressed as a percentage of income (rD). F2CG distributes a fraction DIV/FCF of the annual free cash flow to its shareholders. This is also taxed by dividend taxes, but either way the complete dividend payment will flow back to the public sector. Therefore, all that remains at the financial institutions are the undistributed free cash flows that accumulate as excess cash at the bank accounts of the financial sector (F2CG). Hence, the net interest rate of the public sector can be expressed as follows.

$$r_{net}\left(\frac{D_i}{Y'_i}\right) = \left[\frac{rD_i}{Y'_i} - \varphi\frac{rD_{i-1}}{Y'_i}\right] = \left[\left(\frac{1}{Y'_i}\right)(Income - labour\ costs - corporate\ income\ taxes - dividends)\right] = \left[\frac{rD_i}{Y'_i} - \frac{1}{Y'_i}((1-\alpha)rD_{i-1} + \tau_{CIT}rD_{i-1} + (\alpha-\tau_{CIT})rD_{i-1}(\frac{DIV}{FCF})\right] \quad (5.39)$$

If this summation is approaching its asymptotic debt level (in case of convergence) such that the annual income of the financial sector is almost constant ($r_{net}D_i \approx r_{net}D_{i-1}$) then we can rewrite (5.39) as follows.

$$r_{net}\left(\frac{D_i}{Y'_i}\right) = \left[\frac{rD_i}{Y'_i} - \frac{1}{Y'_i}((1-\alpha)rD_i + \tau_{CIT}rD_i + (\alpha-\tau_{CIT})rD_i(\frac{DIV}{FCF})\right] = r\left(\frac{D_i}{Y'_i}\right)(\alpha-\tau_{CIT})\left(1-\frac{DIV}{FCF}\right)$$

If we divide both sides by (D_i/Y'_i) we get formula (5.40) regarding the net public interest rate.

$$r_{net} = r(\alpha-\tau_{CIT})\left(1-\frac{DIV}{FCF}\right) \quad (5.40)$$

This exactly expresses that what we discussed above – the net interest equals the annual addition to the excess cash of the financial sector which is all income minus all payments to governments (taxes) and households (labour costs and net dividend).

From equation (5.38) we know that if r_{net} is smaller than nominal growth *g* the asymptotic debt level (D/Y) will converge to an asymptotic level of $(g+\Theta)/(g-r_{net})$. We can use equation (5.40) to set the corporate income tax level such that the public debt level will not explode. It must be said though that even without corporate income taxes the debt level will likely remain stable, assuming reasonable values for the average public interest rate *r,* the dividend pay-out ratio DIV/FCF and the added value margin α.

However, if we would conclude that our global financial system is currently stable because public debt relative to GDP approaches a finite asymptotic level we would be jumping into conclusions. We can see this if we split the net public budget constraint into the net governmental budget constraint and the net household budget constraint. Figures 5.6 shows that the public debt converges whereas both the household debt and governmental debt diverge in opposite directions. Figure 5.7 shows the interest payments as a percentage of the respective consumptions (C, G or Y) and displays similar converging and diverging dynamics.

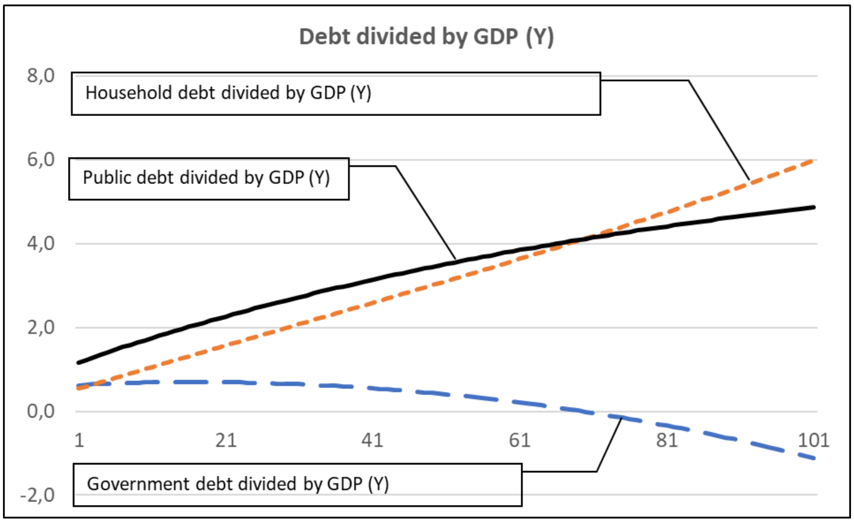

Figure 5.6 Household, Government and Public debt level over a period of 100 years starting at present day debt levels ($D_{Household}$=0.54Y, $D_{Government}$=0.62Y => D=1.16Y)[84]. Input variables assumed constant over time and are specified at table 5.6

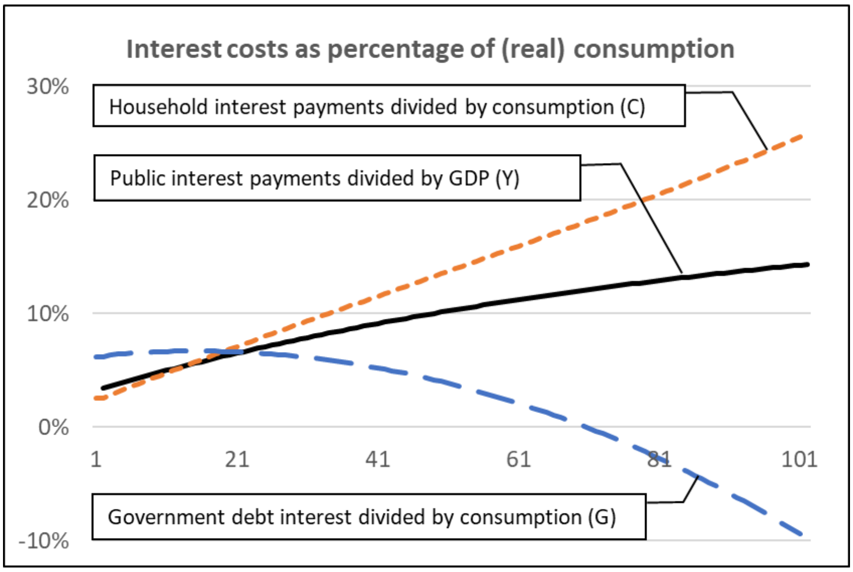

Figure 5.7 Household, Government and Public interest costs relative to respective consumption (either C,G or Y) over a period of 100 years starting at present day debt levels ($D_{Household}=0.54Y_0$, $D_{Government}=0.62Y_0$ => $D=1.16Y_0$)[85]. Input variables assumed constant over time and are specified at table 5.6

[84] https://www.geotrendlines.nl/wereldwijde-schulden-naar-record-247-biljoen/
[85] https://www.geotrendlines.nl/wereldwijde-schulden-naar-record-247-biljoen/

Input variables	Symbol	Formula	Value
Gross Domestic Product	Y (=GDP)	Y=C+G+I	1.000
Nominal GDP growth	*g*	g=(1+Δp)(1+gr)-1	2,0%
Inflation	*Δp*		*2%*
Real economic growth	*g*	g =(1+ΔA)(1+ΔL)-1	*0%*
Reconsumption rate	φ	ΔD/Y-rD=g+(1-φ)	*95%*
Dividend pay out ratio	DIV/FCF	DIVIDEND/FREE CASH FLO	*95%*
Alfa (added value margin)	α	EBITDA=αY	*40%*
Corp. Income Tax rate B2CG	τB2CG	T=τY	*8%*
Corp. Income Tax rate TIB	τTIB	τTIB=τB2CG*(α/(α-s))	*11%*
Labour tax rate	τL	TL=τL*(1-α)Y	*30%*
Dividend tax rate	τDIV	TDIV=τDIV*DIV	*15%*
Investment fraction	s	I=sY	*10%*
Consumption fraction	c	C=cY	*60%*
Government spending fraction	γ	G=γY	*30%*
Interest rate Households	rH		*3%*
Interest rate Governments	rD		*3%*

Table 5.6 Underlying assumptions of figures 5.6 and 5.7. The corporate income tax rates of B2CG (τ_{B2CG}) and F2CG (τ_{F2CG}) are chosen such that the corporate income tax equals ~26% of EBITA for both the financial sector (F2CG) and the real economy (B2CG).

Figures 5.6 shows that even if the total public debt level seems to stabilise, the underlying debt levels can still diverge and make the financial system unstable. This is also shown by Figure 5.7 that tells us that in such an unstable situation, the size of the financial sector (defined as the sum of the absolute values of the consolidated interest flows of both the households and governments) indefinitely outgrows the size of the real economy which results in an ever-increasing financial sector. This is problematic for a number of reasons, but mostly because it draws away highly educated labour from the real economy into the zero-sum game of finance. Table 4.6 shows that this happens even when we choose realistic parameters. The supporting spread sheet (sheet "5.5.6 Financial stability") allows you to vary all parameters presented in table 5.6 over a period of 500 years (annually if you like…).

It is evident that we need to split the various budget constraints and dive into governments and households separately to better understand how to stabilise our financial system. We will do so in the next paragraph (Financial inequality). We will end this paragraph by relating this content to neo-classical growth theory like the Solow-Swan model.

5.5.7 How does all this relate to the "Golden Rule" of neo-classical growth theory?

It is widely known and accepted that the difference between nominal GDP growth and interest (*r-g)* is closely related to financial stability. However, both theory and empirical research mostly focuses on just governmental debt in this regard[86].

From neo-classical growth theories (like the Solow model[87]) on the other hand it appears that a closed economy is stable and runs at full capacity when the interest rate *(r)* is larger than nominal economic growth *(g)*. According to these theories, the optimal state of a closed economy occurs when the interest rate equals

[86] Examples given:
IMF WP/18/82 (https://www.imf.org/~/media/Files/Publications/WP/2018/wp1882.ashx)
OECD Avoiding Debt traps (https://www.oecd.org/eco/growth/avoiding-debt-traps-fiscal-consolidation-financial-backstops-and-structural-reforms.pdf)
Growth in a time of debt, Reinhart/ Rogoff (https://www.nber.org/papers/w15639.pdf)
On the Determination of the Public Debt, R. Barro
(http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.455.8274&rep=rep1&type=pdf)
[87] For details refer to "Intermediate Macroeconomics" by Pablo Kurlat chapter 6
(http://web.stanford.edu/~pkurlat/teaching/Econ%2052%20Notes%202018.pdf)

nominal growth (*r=g)*, which is often referred to as the "Golden Rule"[88]. If *r* is greater than *g*, then the economy is efficient in the sense of having less capital than at the "Golden Rule" steady state"[89].

At first sight equation (5.31) might seem in conflict with neo-classical and endogenous growth theories. However, I believe it is not because there are some important differences. First, in neo-classical growth theory like Solow the interest *(r)* refers to all interest on capital (i.e. ROIC or return on Capital Employed in corporate finance). This includes return on equity whereas in equation (5.31) the interest rate (r) just refers to interest on loans issued to governments and households. It excludes all other consolidated returns, being the free cash flows available for the private sector $(\alpha- \tau_{CIT} -s)Y_i+(\alpha- \tau_{CIT})r_iD_{i-1}$. Therefore equation (5.31) is not necessarily conflicting with The Golden Rule. Consequentially, Solow does not specifically differentiate between loans, money and equity, but instead refers to all as capital. I believe we should distinguish between equity on the one side and loans and money on the other, because equity is protected against inflation whereas loans and money are not.

Another consequence of modelling just one form of capital is that it implies that all savings from investors and employees are automatically and instantaneously reinvested in the real economy (which in our model is referred to as *sY*)[90]. This makes sense in a way that if excess cash is added to capital employed it reduces the returns on capital and hence increases the relative value of labour. On the other hand, in my opinion the Solow model disregards the possibility that money is absorbed from the real economy and is kept still at saving accounts or just used for trading at the aggregated financial markets over long periods of time instead of being used for investing or wage increases. It implicitly assumes that the private sector has no free cash flows, because all income (C+G) that was not spent on operational labour costs is invested in growth. From the FCF on table 5.4 we can derive that Solow's assumption applied to our situation would implicate the following relationship:

$$s_{i+1}Y_{i+1} = (\alpha - \tau_{CIT} - s_i)Y_i + (\alpha - \tau_{CIT})r_iD_{i-1} \ \ for\ any\ given\ i > 0 \quad (5.33)$$

This implicates in our model that the investment rate *s* would not be constant but the investment rate (s_i) would grow every year because companies and banks reinvest 100% of their operational cashflows into capital goods to maximise growth. That way, they annual free cash flow would be 0 for any given year *i* which makes it a self-financing process. This conflicts with mainstream corporate finance theory that dictates that the value of any asset is the net present value of discounted expected future free cashflows. So in my view neo-classical growth theory dictates that for an economy to be efficient, all assets in the economy would be worthless according to the principles of corporate finance, because the investors would never receive any dividend payments.

5.6 Financial inequality

5.6.1 Scope

In this section we will derive the various net budget constraints for governments and households to see (1) what tools they have to reduce budget deficits and (2) to see how well the financial system works for them in terms of benefitting from nominal growth of the real economy and growth of the financial sector. We will focus on the net government budget constraint (5.6.2) and the "net labour income dependent household budget constraint" (5.6.3). The latter is a rather straightforward repetition of the former section (simplified and single government budget constraint). So far, we always assumed wage increases to be in line with nominal economic growth. However, as a matter of fact it appears that wage increases have been lagging nominal growth for the past decades in western economies. Section 5.6.4 discusses two drivers of this and formalises the impact that

[88] Real interest Rate and growth rate by Jean-Marie Le Page (https://ffejournal.files.wordpress.com/2014/11/section-7.pdf)
[89] This quote is taken from a Harvard Paper prepared for the Handbook of Macroeconomics page 48 drafted by Harvard (https://scholar.harvard.edu/files/mankiw/files/govdebt.pdf)
[90] Equation (6.1.3) on page 64 of Intermediate Microeconomics by Pablo Kurlat (http://web.stanford.edu/~pkurlat/teaching/Econ%2052%20Notes%202018.pdf)

lagging wages have on household spending. The final section discusses what I believe are two main drivers of inequality. These are our current system of inflationary fractional reserve banking and inheritance of capital.

I will refer to inequality as financial inequality to stress that it just reflects the registration of wealth distribution, not wealth itself. For example, the average wage adjusted for inflation of an American male worker Joseph Stiglitz refers to is the same as it was 42 years ago implies that the maximum amount of say X breads or Y gallons of gasoline the worker can buy every month remained flat. However, there are all kinds of new products that the worker couldn't buy 42 years ago which he can buy nowadays. Furthermore, with the rise of internet we can now also trade personal data for services. On top, the worker has access to public services that have been improving since (at least in Europe). So, it seems that on average the lives of working people have been improving the past decades despite the fact that wage increases have been lagging registered real economic growth. On the other hand, it seems undeniably true that some people (say the "haves" or the wealthiest 1% or 10% of the western population) at least "on paper" enjoined more income growth the past decades than most other people. Unfortunately, in my opinion this trend will continue untill we change the way we (1) create money (fractional reserve banking) and (2) inherit wealth.

5.6.2 The net government budget constraint

We could split the net public budget constraint and the net interest rate into the net government budget constraint and the net households budget constraint. It is a rather straightforward although nitty-gritty extension of section 4.5.6. Simply put: all labour and net dividends from the financial sector (F2CG) flow back to households. All taxes go to the governments.

So, formula (5.39) still holds and can still be written as follows (we multiplied left and right side by Y'_i and again we assume that the annual debt increase is small such that $D_i \approx D_{i-1}$).

$$r_{net}D_i = \left[rD_i - ((1-\alpha)rD_i + \tau_{CIT}rD_i + (\alpha - \mathrm{s} - \tau_{CIT})rD_i(\tfrac{DIV}{FCF})\right] \quad (5.41)$$

In this formula the various terms on the right side from left to right respectively reflect (1) all interest income of the financial sector, minus (2) labour costs of all bankers, (3) corporate income tax charged to the financial sector and (4) dividend payments from the financial sector. We would need to incorporate (5.41) into the exact public budget constraint and then separate all fund flows into a household circuit and a government circuit. The complexity lies in the fact that the governmental income and household income are interdependent in a way that the government receives taxes over consumer spending and rental income from household debt and households receive dividends from government spending and rental income from government debt. It would yield a long complex (i.e. many interrelated terms) equation. Also, unless the interest on government debt and household debt are equal (which is not the case in real life) the net public interest rate r_{net} cannot be constant over time. This is because it is the weighted average of the net interest rates of household debt and government debt which are not growing at equal pace.

Alternatively, we could just start with the exact governmental budget constraint and work from there. To do so, assume the government raises dividend taxes of τ_{DIV} percent on all dividend payments and labour income tax of τ_L percent of all labour income. Also we need to distinguish between household interest costs *($r^H_{net}D^H_i$)* on the one side and government interest costs *($r^G_{net}D^G_i$),* that jointly adds up to *($r_{net}D_i$).*

The government budget constraint is given by equation (5.42), wherein T_{i+1} represents all tax income in year *i+1*.

$$\Delta D^G_{i+1} = G_{i+1} + r^G D^G_i - T_{i+1} \quad (5.42)$$

In words this means that the budget deficit (*ΔD*) in year *i+1* equals all government spending *(G)* added with the interest costs *(rD)* minus all tax income *(T)* in year *i+1.*

This can be rewritten as follows:

$$\Delta D^G_{i+1} = [G_{i+1}]_{Real\ economy\ spending\ of\ government(s)}$$

$$+ [r^G D_i^G]_{interest\ costs\ of\ government(s)}$$

$$- \left[\tau_{CIT} Y_i + \tau_L (1-\alpha) Y_i + \tau_{DIV} \left(\frac{DIV}{FCF}\right) (\alpha - \tau_{CIT} - s) Y_i\right]_{Real\ economy\ tax\ income}$$

$$- \left[\tau_{CIT} (r_i D_i) + \tau_L (1-\alpha)(r_i D_i) + \tau_{DIV} \left(\frac{DIV}{FCF}\right) (\alpha - \tau_{CIT})(r_i D_i)\right]_{Financial\ tax\ income} \quad (5.43)$$

This formula doesn't look very pleasant, but essentially is very simple. It subsequently describes two components of spending (spending *G* in the real economy and *rD* of interest cost) and two components of income, which are (1) income from taxes raised on the real economy and (2) income from taxes raised on the financial sector. In both cases it describes (1) corporate income taxes, (2) labour income taxes and (3) dividend taxes. Please note that this equation is still incomplete as it ignores income from taxes raised on capital. We will get to this later.

In formula 5.43 D_i represents the sum of D^G_i and D^H_i and *r* represents the weighted average interest rate of household interest (r^H) and governmental interest (r^G), which as mentioned before is no longer constant unless the interest rates on household debt (r^H) and government debt (r^G) are equal.

Please note that we disregard private debt in this book since we eliminated all transactions and positions between companies and financial institutions when consolidating the private sector. We could easily include private debt into equation (5.43) by assuming that D_i is the sum of all debt in the closed economy including the debt of all companies (D^{B2CG}_i) and the debt of all banks (D^{F2CG}_i) between each other. In formula this is:

$$r_i D_i = r^H D_i^H + r^G D_i^G + r^{B2CG} D_i^{B2CG} + r^{F2CG} D_i^{F2CG} \quad (5.44)$$

Including definition (5.44) that includes private debts into the governmental budget constraint (5.43) would hold true for any closed economy.

Finally we could assume that (5.43) represents the budget constraint of a single government acting in an open economy. This way, formula (5.44) describes all debt and rental income raised by financial institutions (either nationally and internationally) under the jurisdiction of the government. The GDP (Y_i) would then be described as formula (5.4).

$$Y_i = GDP_i = C_i + I_i + G_i + (X_i - M_i) \quad (5.4)$$

Now, for the real maths-fetishists we can divide both sides of formula (5.44) by Y_i and assuming constant nominal growth *g* for all components C_i, I_i, G_i, X_i and M_i of the GDP (Y_i) to obtain (5.46), with Υ being the fraction of the government spending *G* of the GDP (Y_i):

$$\frac{\Delta D_{i+1}^G}{Y_i} = [\gamma(1+g)]_{Funding\ required\ to\ adjust\ government\ spending\ for\ nominal\ growth}$$

$$+ \left[\frac{r^G D_i^G}{Y_i}\right]_{governmental\ interest\ costs\ as\ percentage\ of\ GDP}$$

$$- \left[\tau_{CIT} + \tau_L (1-\alpha) + \tau_{DIV} \left(\frac{DIV}{FCF}\right) (\alpha - \tau_{CIT} - s)\right]_{Real\ economy\ tax\ income\ as\ \%\ of\ GDP}$$

$$- \left[\tau_{CIT} \left(\frac{r_i D_i}{Y_i}\right) + \tau_L (1-\alpha) \left(\frac{r_i D_i}{Y_i}\right) + \tau_{DIV} \left(\frac{DIV}{FCF}\right) (\alpha - \tau_{CIT}) \left(\frac{r_i D_i}{Y_i}\right)\right]_{Financial\ tax\ income\ as\ \%\ of\ GDP} \quad (5.45)$$

Rearranging the various terms and substituting yields formula (5.46).

$$\left[\frac{\Delta D_{i+1}^G}{Y_i}\right]_{\frac{\Delta D_{i+1}}{Y'_i}} = [\gamma g]_g + \left[\gamma - \tau_{CIT} + \tau_L (1-\alpha) + \tau_{DIV} \left(\frac{DIV}{FCF}\right) (\alpha - \tau_{CIT} - s)\right]_{(1-\varphi)}$$

$$+ \left[\left(\frac{r^G D_i^G}{Y_i}\right) - \left(\tau_{CIT} \left(\frac{r_i D_i}{Y_i}\right) + \tau_L (1-\alpha) \left(\frac{r_i D_i}{Y_i}\right) + \tau_{DIV} \left(\frac{DIV}{FCF}\right) (\alpha - \tau_{CIT}) \left(\frac{r_i D_i}{Y_i}\right)\right)\right]_{r_{net} D_i / Y_i} \quad (5.46)$$

This essentially is the net government budget constraint for a government in an open economy denoted as a percentage of GDP with a form equal to formula (5.37). To make the comparison easier the various terms of (5.46) are flagged grey with the equivalent terms of (5.37).

$$\frac{\Delta D_{i+1}}{Y'_i} = g + (1-\varphi) + r_{net}\frac{D_i}{Y'_i} \quad (5.37)$$

In this formula (5.37) φ represents the fraction of government spending in the real economy that is regained by raising taxes from companies (corporate income taxes), employees (labour taxes) and investors (dividend taxes). The net interest rate is financial spending of the government (i.e. the interest payments on government debt) minus the amount of taxes raised from the financial sector (again these are corporate income taxes, labour taxes and dividend taxes), expressed as a fraction of the government debt.

We just derived the government budget constraint for a single government in an open truly capitalistic economy with a global fractional reserve banking system (ignoring capital taxes). This took some dull and nitty-gritty efforts, but I believe it was worth it. We can now use this formula to see which governments are well-positioned or ill-positioned to maintain prudent budgeting.

The first terms at the right side of the equation *(γg)* denoted as percentage of Y_i is the amount of funding the government needs to borrow in order to adjust its spending to real economic growth and inflation. If the government wants to reduce its budget deficit it could decide to reduce its spending. This will have a material impact on gross GDP growth though, since government spending *(G)* typically amounts roughly 30% of GDP. So, a 1% reduction in government spending has a negative impact of 30% times 10% is 0.3% on GDP. The positive impact on the budget deficit of the government will be less than 1%, because the government will miss the taxes that otherwise would have been raised on the real economy over the 1% government spending the year thereafter (and so on). Typically, this is 30%[91] over the spending reduction which reduces the 1% cost cutting into a (1-30%)*1% ≈ 0.67% effective reduction of the budget deficit. In general the larger the size of the government, the larger the economic dependence of the economy, the less effective the measure will be. So, the measurement of reducing government spending will be most attractive in case of economies with a trading surplus that have a small government. Exactly the opposite of the type of economies that are likely to have a government with problematic budget deficits.

The second option is to raise taxes implied on the real economy. This could be corporate income taxes, labour income taxes or dividend taxes. Increasing labour income taxes has the largest impact on the economy because it reduces the household consumption capacity which typically amounts to 60% of GDP. However, it would not further drive income inequality, because $\alpha_{PIKETTY}$ will remain unchanged. Again, economies with a trading surplus are more resilient to raising labour income taxes.

Raising corporate income taxes often is difficult for governments in today's competitive international business climate. Nonetheless the benefits would be that company investments *(I=sY)* typically amount to just 10% of GDP. So even if the lower net profitability of companies would reduce corporate investments the impact on GDP would be limited. Equal to any other fiscal measure, the benefits for governments of economies with a trading surplus are larger and these economies are also more resilient to the negative impact of these measures. This is because company's investment decisions are largely based on expected future profitability, so an internationally oriented economy with a trading surplus will have a lot of exporting companies which are less likely to reduce its local investments based on a local CIT raise.

Raising dividend taxes might be the measure that generally has the least impact on future GDP. This is because it generally hits households that have budget surpluses. Obviously, it hits pension funds, insurance companies and alike, but this does not immediately impact the pensions and insurance claims that flow back into the real economy. Unfortunately, dividend taxes on real economy business cover only a small fraction of the government budget. So dividend taxes must be raised significantly to have any material impact.

[91] This is typically 30% either through 25% of EBITA followed by some dividend taxes or 30% labour income tax over missed labour income due to labour income.

The third set terms of terms (denoted by $r_{net}D_i/Y_i$) describes the opportunities of the government to tax the financial sector. Obviously, the various options are equal in case of taxing the real economy, so governments of economies that have a large internationally oriented financial sector (e.g. UK or Switzerland) have an advantage compared to governments of countries with just a small (national) financial sector. Unlike the case regarding countries with strong real economies, in case of an international financial sector wealth is just transferred from other nations to the citizens of countries that have banks that finance these activities. Secondly it shows that governments in fact benefit if private and household debts are large within the economy since it allows them to raise more taxes over the financial sector.

Summarising it seems that raising taxes on the real economy in all cases comes at a price, which is larger for importing economies than for exporting economies. This all should be no surprise, because finance is a zero-sum game. Somebody's gain is someone else's loss. This is no different for importing economies that raise import taxes in an attempt to reduce budget deficits. Somebody (either local companies and consumers or exporting nations) always pays. Secondly, formula (5.46) shows that governments benefit if the economy has a large financial sector, which provides more opportunities to raise taxes, especially in case of an internationally oriented financial sector. This is because when these banks finance ventures in other countries, these banks will receive interest payments which can be taxed. So, international banking essentially transfers wealth from one nation to another, hence the benefits of having a large international financial industry.

Governments of importing nations that have no significant financial sector will likely have a hard time maintaining prudent budgeting in today's international competitive business climate. If everything else fails, these governments still have 2 options left, being (1) raising import tariffs or (2) increasing inflation such that the government debt gradually inflates away. Now imagine a government that is also part of a monetary union and a free trade union such that they no longer control the money supply and can not raise import tariffs. Nor are they entitled to extraordinary amounts of natural resources. Apart from taxing capital or formalising the informal and black economy, these governments have no option but to force their citizens to work harder for less income over long periods of time. Examples of such governments are Italy, Greece and Spain. No wonder young educated people leave those countries to build a career elsewhere.

So, at the end of the day some governments are better equipped to maintain prudent budgeting than others. The governments that need to raise taxes or cut spending in order to maintain prudent budgeting are the exact governments that are ill positioned to do so. This is because any increase in taxes or decrease in spending can result in an outflow of capital and labour in today's competitive international business climate. For governments this makes their financial situation fragile, especially if they have no authority on the money supply which rules out inflationary debt reduction.

5.6.3 The net labour income dependent households budget constraint

Imagine a group of all households in a closed truly capitalistic economy that are fully dependent on income from labour that annually spends a constant fraction c^L of the total GDP (Y_i) for any given year i such that their joint total consumption $C^L_i = c^L Y_i$ for any given year i. Let's also assume that their joint annual net income L^L_i is annually adjusted for inflation and real growth such that L^L_i is a constant fraction α^L of the GDP (Y_i). This way the budget constraint of all the households that fully depend on income from labour in a closed economy is as follows:

$$C^L_{i+1} + rD^L_i = L^L_i + \Delta D^L_{i+1} \qquad (5.47)$$

In words this means that the total consumption of the group of households that fully depend on labour income plus their joint interest payments must equal their income from the prior period plus the amount of new debt they need to borrow to fund the gap. We could rearrange this equation as follows:

$$c^L Y_{i+1} + rD^L_i = \alpha^L Y_i + \Delta D^L_{i+1} \qquad (5.48)$$

Now assume that c^L equals α^L. This implies that these households maintain their level of consumption constant adjusted for inflation and real growth and fund this with their income from the year before plus an amount they borrow to cover the nominal growth and interest payments every year. This assumes that all households that fully depend on income from labour constantly spend more than they earn to maintain their level of consumption. This might seem unrealistic, but I believe (without having checked it) this is actually what has been happening for decades in case of large amounts of households. The reason is that we know for a fact that the total public sector inevitably spends more than they earn (due to our system of fractional reserve banking and central bank policies to inflate at roughly 2% annually) and it seems unlikely that the group of households that enjoys income from capital drives all nominal consumption growth. This is because they are much richer on average and do not need the annual growth in their income (that outgrows the pace of economic growth because it is partly income from capital). They are the savers. Off course governments could pick up all consumption growth, but this implies that their consumption growth would be roughly 3 times the nominal GDP growth (because it would have to make up for the lack of consumption growth which is typically double the size of government spending). What I believe happens is that the households that fully depend on labour income (let's say the average people) which is the vast majority of the households in western economies is constantly stimulated by both the private sector, the central banks and governments to overspend. The main drivers that trigger structural household overspending are probably:

- Continuously increasing valuations of real estate owned by households driven by an ever-increasing amount of inactive money in the financial markets that can be used for leveraging (or makes people feel richer than they actually are).
- Increasing supply of consumer credit facilities;
- Increasing supply of private leasing facilities; and
- Increasing supply of pay-per-use of capital goods.

In any case we could redefine the group of households such that it exactly contains all households that fully depend on labour income and constantly borrow money to (1) cover nominal growth and (2) pay their interest obligations such that they can enjoy a level of consumption that grows in line with real economic growth over longer periods of time *($c^L=\alpha^L$)*. The budget constraint for this group of households equals formula (5.48) which can be written as follows:

$$\alpha^L Y_{i+1} + rD_i^L = \alpha^L Y_i + \Delta D_{i+1}^L$$

This equals:

$$\alpha^L (1+g) Y_i + rD_i^L = \alpha^L Y_i + \Delta D_{i+1}^L$$

Which can be rearranged as equation (5.49).

$$\frac{\Delta D_{i+1}^L}{\alpha^L Y_i} = g + \frac{rD_i^L}{\alpha^L Y_i} \qquad (5.49)$$

The equivalent continuous time differential equation with *$x(0)=0$* wherein *$x(t)=D^L(t)/\alpha^L Y(t)$* is the amount of debt of this group of households relative to their joint net income has the following solution:

$$x(t) = \frac{D^L(t)}{\alpha^L Y(t)} = \frac{g}{g-r}\left(1 - e^{(r-g)t}\right) \qquad (5.50)$$

Obviously, equation (5.50) disregards defaulting and holds true only for households that fully depend on income from labour and consistently fund nominal growth and interest obligations by borrowing money. Nonetheless it reveals the main difference between households that fully depend on income and the rest of the public sector (i.e governments and households that own equity). They are unexposed to financial income. Therefore, they have no feedback loop at all from their interest payments. Consequently, their net interest rate *(r_{net})* equals the interest rate they pay for their loans. In order to maintain a sustainable financial position, the interest rate theses households pay must be lower than nominal growth. And we all know that this is not the case. Even interest rates on mortgage-backed securities generally exceed nominal growth, let alone all other forms of consumer credit. This makes these households vulnerable when borrowing.

In today's capitalism-based economies central banks and the private sector are so focused on (GDP) growth that they will do anything to maintain nominal GDP growth. The result is that the public sector consistently spends more than they earn.

Equation (5.50) captures the essence of this debt-financed growth; for labour income dependent households this way of financing growth is not sustainable unless debt is for free without repayment obligations, which would make it a gift rather than a loan.

So although on aggregated level the total household debt levels seems stable there will inevitably always be households that fully depend on labour and have an income development on or below nominal GDP growth. These households face decreasing real spending capacity over time or they inevitably will default sooner or later (or die indebted).

5.6.4 Household spending when wage increases are lagging nominal growth

Introduction

Obviously, the assumption that there exists a large group of households that structurally spends more than they receive should be substantiated by empirical data. On the other hand, equation (5.50) assumes wages that grow in line with nominal growth. This is a totally unreasonable assumption. In the USA "from 1979 onwards real hourly wages for the great majority of American workers have stagnated or even fallen"[92] Also in Europe wages have been lagging nominal economic growth for decades. We will include the consequences for people that depend on labour income by including lagging wage development into equation (5.50) later on, but first we will briefly touch upon what I believe are the most important drivers of lagging wage developments which yields to Piketty's claim that return on capital growth exceeds nominal GDP growth ("$r_{capital} > g$").

Piketty's research is largely empirical. It is beyond the scope of his research to explain why "$r_{capital} > g$". I am confident that at least in the past decades fiscal regimes have become increasingly "company friendly" and secondly outsourcing to low-cost-countries replaces jobs away from western economies. The latter driver can be ignored when observing a closed economy. So, part of the increase probably is a result of these factors. Fiscal reasons should be a manageable problem, save aside growing international competition between fiscal regimes to attract companies. Other than fiscality, I believe there are two underlying drivers of lagging wages, which are (1) the dominance of the idea that companies should "maximise shareholder value" in the international business environment and (2) lagging response of the labour supply to continuous labour productivity increase (automation).

Maximising long-term shareholder value

Mainstream corporate finance like the net present value method dictates that the value of a company is exclusively determined by the expected future free cash flows discounted by an applicable discount rate (like formula 4.1)[93]. The other main dogma is that the primary objective for any company should be to "maximise long-term shareholder value"[94]. Combining both principles of corporate finance reveals that the private sector of a capitalism-based economy has a direct objective to maximise income inequality ($\alpha_{Piketty}$). See for example the FCF of the private sector in table 5.4. This is mathematically expressed by formula (5.51).

$$Value = \frac{Free\ Cash\ Flow_{t=1}}{WACC-g} = \frac{(\alpha-\tau_{CIT}-s)Y_1}{WACC-g} \cong \frac{(\alpha_{Piketty}-s)Y_1}{WACC-g} \qquad (5.51)$$

The formula disregards the cashflows of financial institutions. Needless to say that this is no different.

[92]Quote taken from "The Value of Everything" by Mariana Mazzacuto (Penguin Books 2018).
[93] Formula 3.1 shows that for any given cost of capital (WACC) the value of an unlevered company is exclusively driven by the future free cash flows the company is expected to generate.
[94] Koller e.a. for example states on page 4 that "Managers and board-members…should set long-term shareholder value creation as their primary objective"

In other words, companies that adopt mainstream corporate finance implicitly also adopt increasing income-inequality as their primary objective. The way to do this is to develop an **earning model**. Like the purpose of a strategy is to optimise the variables *s, Y, WACC* (disregarding capital structure) and *g,* the sole purpose of any earning model is to maximise the profit margins which aggregates into α. In other words, companies aim to capture as much of the value they create for the account of the shareholders. Unlike a strategy this is no longer about creation of wealth but is about distribution of wealth. And because distribution of wealth is a zero-sum-game the success of any company's earning models will always be at the cost of the other stakeholders. This can off course be other companies (and its shareholders), but in case of a private sector that aims to maximise α this must be either employees, governments or consumers. The way to maximise profits is to reduce or eliminate exposure to the dynamics of well-functioning markets (avoid perfect competition).

So, if companies are successful in avoiding perfect competition by raising barriers and switching costs the argument of free markets no longer holds and companies will capture value for the benefit of their shareholders at the cost of other stakeholders. Table 4.6 shows an overview of common earning models and relates them to the conditions of free markets these earning models are aiming to mitigate.

Condition for perfect competition	**Common earning model**
Homogeneous products	Innovate, differentiate, be unique Stay away from commodities unless you are cost-leader
No barriers to entry or exit	Create entry barriers, switching costs and customer lock-in
Every participant is a price taker. No participant with market power to set prices	Become market leader, be dominant, set the standards
Rational buyers	Marketing and branding, "perception is truth"
No externalities or government intervention	Lobbying, create favourable government intervention and protective regulation
No economies of scale	Create economies of scale
Network effects ensures sufficient supply	Be scarce and exclusive
Perfect information	Exploit information asymmetry

Table 5.7 Conditions for perfect competition often conflict with earning model strategies

You will notice that the list of conditions for perfect competition is the exact opposite of the list that management books typically will provide as strategies for margin improvement[95].

Growth is driven by innovation, but limited by education

Real economic growth is exclusively driven by labour productivity increase and labour population growth. In neoclassical growth theory these components are usually denoted as $A(t)$ and $L(t)$ respectively[96].

Labour productivity growth (which equals growth of output per worker) can be (1) efficiency gains or (2) automation. The first driver refers to organisational improvements like planning and optimising distribution chains. The second driver is the development of machinery and tools to leverage labour productivity. Undeniably automation is an enormous driver in today's global labour productivity increase. Think of machine learning, artificial intelligence, robotics and alike. In economics we generally believe that output per worker evolves linear. This is based on research which is commonly referred to as the "Kaldor Facts". Among other things this empirical research concludes that "the rate of growth of output per worker is constant over long periods of time"[97] or "the rate of growth of GDP per capita is constant" [98] . This would be an exception then. In both physics and economics things always seem to evolve exponentially. Why should automation be any

[95] https://en.wikipedia.org/wiki/Perfect_competition
[96] See for example page 81 equation (6.5.1) of Intermediate Macroeconomics by Pablo Kurlat (http://web.stanford.edu/~pkurlat/teaching/Econ%2052%20Notes%202018.pdf).
[97] https://en.wikipedia.org/wiki/Kaldor%27s_facts bullet 3.
[98] Page 55 of Intermediate Macroeconomics by Pablo Kurlat (http://web.stanford.edu/~pkurlat/teaching/Econ%2052%20Notes%202018.pdf) or

different? Is it not reasonable to assume that automation growth is correlated to Moore's Law?[99] It might be early days to detect, but I wouldn't be surprised if a few decades from now we will have alternative Kaldor Facts.

Either way, I believe automation is the other driver of income inequality. At least on a global scale, which obviously rules out outsourcing production to low-labour-costs-countries, it is.

Historically automation gradually evolved (either linear or exponentially) from taking over the simplest human and animal tasks to increasingly complex human tasks. In the middle ages automation could for example be a windmill grinding grain or pumping water to create new land. Or it could be a simple plough behind an ox. From there humans developed some more complex tools like looms and book presses all the way to today's development into artificial intelligence, internet of things, self-driving cars and alike. While doing so, automation gradually shifts the demand curve of labour towards increasingly high-level labour (and will continue to do so). Accordingly, the supply curve of labour needs to adept. This means that we continuously need to train and re-educate our labour pool to keep up with an ever-evolving demand towards jobs with increasing complexity or jobs that require different skillsets. This process is visualised in figure 5.8, wherein the vector lengths denoted by dL_D/dt and dL_S/dt respectively represent the pace of the demand curve of labour (L_D) driven by the pace of automation and the pace of the supply curve (L_S), driven by our ability to adept our labour pool to the shifting demand. The figure shows a normal distribution, but the curves could equally likely have any other shape (with coinciding mean and maximum) and should not necessarily be symmetric or equal to one another.

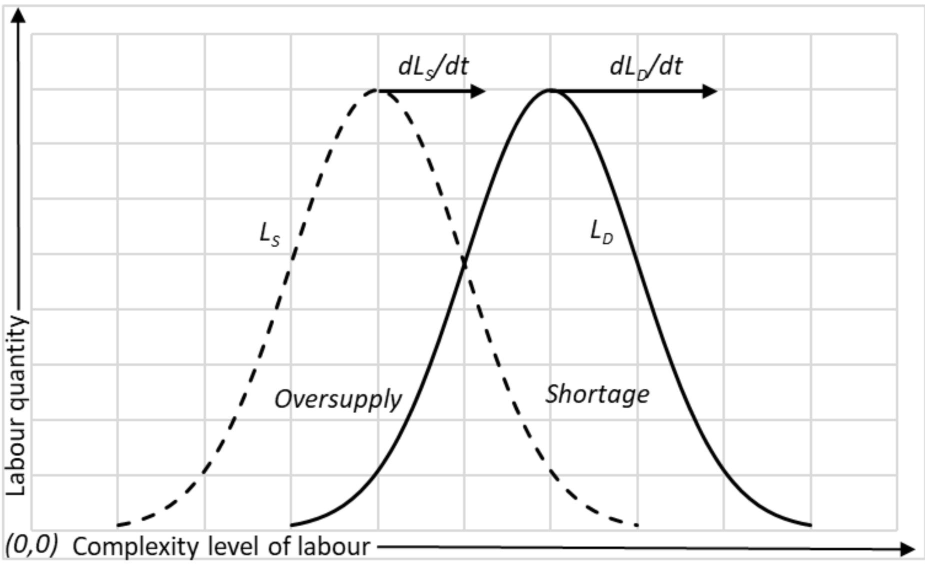

Figure 5.8 Labour supply curve (L_S) lagging labour demand curve (L_D)

Figure 5.8 shows two things. First, we will have to accept that the supply curve will always lag the demand curve. After all, we can only (re-)educate our labour pool to fit the demand of an innovation once the capital good (machine or software) is invented. Otherwise, how are we supposed to know what to teach employees if the thing that needs to be operated is not yet invented. So, the best we can do is to create a (re-) educational system that is flexible and fast, to minimise the gap between supply and demand. However, we also must except that over a longer period the pace of (re-)education can never exceed the pace of innovation ($dL_S/dt < dL_D/dt$). To meet the shifting demand, we also continuously need to (re-)educate our labour pool accordingly. Otherwise, newly invented capital goods would remain unstaffed. So, although growth is driven by automation, it is also limited by our ability to adept our labour pool accordingly (which is about education). In other words,

[99] https://en.wikipedia.org/wiki/Moore%27s_law

economic growth is driven by our ability to create new material capital goods (demand curve) but limited by our ability to create immaterial capital goods (like human skills and expertise).

Secondly, figure 5.8 tells us that because automation replaces human jobs this changes the demand curve of labour and hence impacts the labour markets. The curves assume that demand of labour disappears at the lower end of the demand curve. That way the employees in that segment will suffer from decreasing demand that eventually vanishes completely. This implies that employees that can only perform these jobs are at that point in time economically worthless. Hence in that case free markets and perfect competition theories do not apply to this situation anymore. Like we do not use horses anymore in agriculture. Even if they were free to use. Regardless what jobs are automated the employees involved will suffer from a deteriorating negotiating position and will have to settle for lower paid jobs at best. Now, what we can learn from companies is that we should develop a strategy and an earning model to make things better for these people. Obviously, this is all about centrally coordinated re-education. This would also be in the best interest of companies, because rejecting any responsibility in this regard removes any incentives of employees to be open about their personal contribution on the company's performance[100]. Secondly, it would reduce their growth rate because capital goods remain understaffed.

The labour income household constraint with lagging wage development

Now let's include lagging wage development into our net labour income dependent households budget constraint as expressed by equation (5.47).

$$C_{i+1}^{L} + rD_{i}^{L} = L_{i}^{L} + \Delta D_{i+1}^{L} \tag{5.47}$$

Now assume wages grow at a slower pace *(g_L)* than nominal economic growth *(g)*, such that $g_L < g$. Obviously, α^L is no longer constant over time, but will grow driven by increasing profit margins. If households that fully depend on labour income earned a fraction $\alpha^L{}_0$ of Y_0 in year 0, then their income will develop over time as described by equation (5.52).

$$L_{i}^{L} = \alpha_0^L Y_0 (1+g^L)^i = \alpha_0^L (1+g^L)^i \left(\frac{Y_i}{(1+g)^i}\right) = \alpha_0^L Y_i \left(\frac{1+g^L}{1+g}\right)^i \tag{5.52}$$

If these households consumed a fraction $\alpha^L{}_0$ of Y_0 in year *0* and increase their annual consumption with nominal growth *g* their budget constraints is given by equation (5.53).

$$\alpha_0^L Y_i (1+g) + rD_i^L = \alpha_0^L Y_i \left(\frac{1+g^L}{1+g}\right)^i + \Delta D_{i+1}^L \tag{5.53}$$

We can rearrange this into formula (5.54).

$$\frac{\Delta D_{i+1}^L}{\alpha_0^L Y_i} = g + r\frac{D_i^L}{\alpha_0^L Y_i} + \left[1 - \left(\frac{1+g^L}{1+g}\right)^i\right] \tag{5.54}$$

I have no clue how to solve this, but it clearly shows that this equation also contains a term that dictates the amount households need to borrow to cover the gap in wage increase if $g^L < g$, which increases as time passes. Please note that in case that wage increases grow in line with nominal economic growth we obtain equation (5.49) again. Needless to say that households that are exposed to a budget constraint like (5.54) are even more vulnerable and will sooner or later inevitably lower their consumption pattern to meet their financial obligations. A sheet called "5.6 Financial inequality" allows users to assume differing growth rates of GDP and wages to see the impact this has on inequality and financial stability.

Summarising it seems that like vulnerable governments we also have vulnerable households in a closed truly capitalistic economy. These are households that fully depend on labour and have little (career) opportunities to increase their income. These households should grow their spending below nominal growth, even if their

[100] David Greaber (Bullshit Jobs) concludes that roughly 25% of the employees admit that their job is useless

income increases with nominal economic growth. Otherwise they will be caught in an unsustainable Ponzi-scheme.

5.6.5 Fractional reserve banking and inheritance are main drivers of inequality

Introducing β

There are two ways to look at inequality. You could look at income or at capital. We will refer to inequality of income as α-inequality (referring to the balance between in income on labour and income on capital as defined by formula 4.7). We will refer to inequality of capital as β-inequality. This refers to the metric *β* that is commonly used to relate the aggregated amount of capital of an economy to the corresponding NDP *(Y')* like formula (4.30)[101]. Here *K* is defined by formula (3.1) as the sum of both equity and money (loans are netted against debt).

$$\beta_i = \frac{K_i}{Y_i} \quad 5.55$$

Both definitions of inequality are much debated and criticised[102] and have pro's and cons'.

Piketty conducted empirical research in most western societies in recent history (starting in roughly 1800 in France which seems to have the best historic data available). This research shows that except during the world wars and the period in between both α and β have been structurally increasing. Currently, western mature economies have an α of roughly 25% and a β between 5 and 6 (based on NDP). Piketty's main conclusion is that return on capital is structurally larger than nominal economic growth (a phenomenon he refers to as *"r>g"*) which is the main driver of increasing β inequality.

By definition *β* does not take the amount of debt in the economy into consideration, because for every loan provided there exists an equal negative amount such that they eliminate each other. However, when it comes to measuring inequality I believe it is better to include public debt, because (1) it is an indicator of the claim on value that the private sector (central ownership) has on society (the public domain) and (2) total public debt can be a significant relative to the size of the money supply *(M)* and all equity *(E)*. So, we will define another metric denoted by β^D that includes the amount of public debt in the financial system

$$\beta_i^D = \frac{K_i + D_i}{Y_i} \quad 5.56$$

In equation (5.56) D_i represents the total amount of public debt in the system. The definition purposely excluded private debt, because this is a claim within the private sector and hence does not impact inequality between households that own equity and households that do not. Unless stated otherwise, we will assume the value of all equity in the economy (E_i) is based on market value.

Increasing capital Inequality (β^D-inequality) I believe is fundamentally driven by two things. These are (1) our system of fractional reserve banking to create money along with central banks' policies of inflation (referred to as **"inflationary fractional reserve banking"**) and (2) intergenerational transfer of wealth (or inheritance).

1. **Inflationary fractional reserve banking comes at a price**

I might be wrong on the details, but in general there are two ways central banks like The Federal Reserve and the ECB normally intervene in the financial system. They can (1) influence the interest rates and (2) intervene in the money supply.

1. **Interest rate** - Central banks set the interest rates they charge to (or pay on) on the loans (debt) the central bank provides (holds) to commercial banks at any rate they liked at their sole discretion. In general central banks will lower the interest rates when economic growth (and inflation) is low. This

[101] This is the definition Piketty uses (see for example page 67-73)
[102] https://www.cnbc.com/2018/01/22/wef-18-oxfam-says-worlds-richest-1-percent-get-82-percent-of-the-wealth.html

way, the commercial banks (F2CG) can for example reduce the interest rate they charge to commercial banks in an attempt to reduce the interest rates that commercial banks charge to companies and households.

2. **Control of the money supply *(M_i)*** – The second variable that they control is the money supply in the financial system *(M_i or just M)*. To do this so they have three options.
    a. **Fractional Reserve Banking** – commercial banks can create money by providing loans to consumers and companies up to a level called the "reserve requirement". This is the percentage of loans provided they need to keep liquid (i.e. immediately available on request) to be able to meet demand from people and companies who want to withdraw from there saving accounts. If the central bank lowers the "reserve requirement", say from 10% to 8%, banks can issue more loans and hence money is created. If the central bank wants to reduce the money supply, they just raise the "reserve requirement" which gradually reduces the money supply as loans are repaid and banks can no longer re-issue loans until the meet the new requirement;
    b. **Open Market Operations** (OMO) – Next to adjusting the interest rate banks commonly issue (or buy) government bonds to or buy from commercial banks with a goal to influence the interest rate and regulate the money supply.
    c. **Quantitative Easing –** Thirdly central banks have the option of quantitative easing, which is commonly referred to as "printing money" and considered an emergency measure. The central bank injects money into the financial system at a large scale by buying assets from the capital markets (through the commercial banking system) such as portfolios of mortgage backed securities (e.g. a set of loans provided to homeowners with a pledge on the houses acquired) or again governments bonds. For some reason, the central bank do not acquire equity though.

Now, from the exact public budget constraint formulas (5.23) to (5.25) we know that governments and consumers at aggregated level in a closed truly capitalistic economy that is steadily growing at nominal rate *g* on average must borrow money from the private sector every year to fund (1) their nominal consumption growth *g*, (2) the gap that some households save *(1-φ)* and (3) their interest requirements *(rD/Y')*, which (4) can largely be regained by governments but not by households that depend on income (φrD/Y').

$$\frac{\Delta D_{i+1}}{Y'_i} = g + (1-\varphi) + \left[r\frac{D_i}{Y'_i} - \varphi r \frac{D_{i-1}}{Y'_i} \right] \qquad (5.23)$$

Western central banks (including the Federal Reserve, European Central Bank, Bank of England) mostly create money by Fractional Reserve Banking. This means that within certain restrictions creation of money is (largely) delegated to commercial banks. Therefore, the public sector has no choice but to constantly borrow money from the private sector to fund their deficits as expressed by equation (5.23). Disregarding default and capital taxes (that for some reason are a negligible part of the tax income of governments) governments and households have no choice, but to cut their spending budgets if they wish or are required to reduce their debt. However, the consequence at consolidated level will inevitably be that household consumption *(C)* and/or government spending *(G)* will decline. Because these components comprise roughly 90% of GDP (Y=C+G+I) the economy would then enter into a period of negative nominal growth, unless the rich households fill this gap by increasing their consumption such that φ becomes larger than 1. This seems very unrealistic, because their additional consumption will largely be in the field of luxury goods. And to the extent the savings are owned by average families they would be spending their retirement savings. So, the only way to reduce public debt is to go through (what economists would call) a period of recession, which is negative GDP growth. Probably this will come with deflation as companies compete for a reduced demand. For mainstream economists, central bankers and policy makers both negative GDP growth and deflation are totally unacceptable (not to say their worst nightmare), even if GDP growth has become completely disconnected from real economic growth. The common way to fight negative GDP growth is to aim for an increase of the money supply above real economic growth, which eventually should result in inflation. Normally central banks will first lower the interest rates they charge to commercial banks in an attempt to reduce commercial interest rates such that households, companies and governments are tempted to borrow money for consumption. If that doesn't work, their second and more disturbing method is called Quantitative Easing. This way central banks buy government bonds and other securities at the financial markets (at large volumes) and as such inject money into the aggregated financial markets an withdraw securities. However, from what I know there already exists a surplus of money in the financial markets (structurally saved by companies and wealthy households, previously referred to as 1-φ). The money shortage occurs in the real economy. quantitative easing increases the (inactive) money supply into the financial markets and withdraws securities from financial markets. My guess is that quantitative easing unnaturally drives valuation of financial securities more than it drives inflation in the real economy. And besides, if central banks want to intervene on the financial markets, why do they buy low grade uncovered loans (usually issued by highly indebted governments) instead of equity. For example, since 2015 the ECB injected EUR 2.600 billion into the financial markets by buying debt that they now have on their balance sheet. They could have bought significant stakes in European major listed companies which would have exposed European citizens to a stable dividend income stream and to co-ownership of real underlying capital goods.

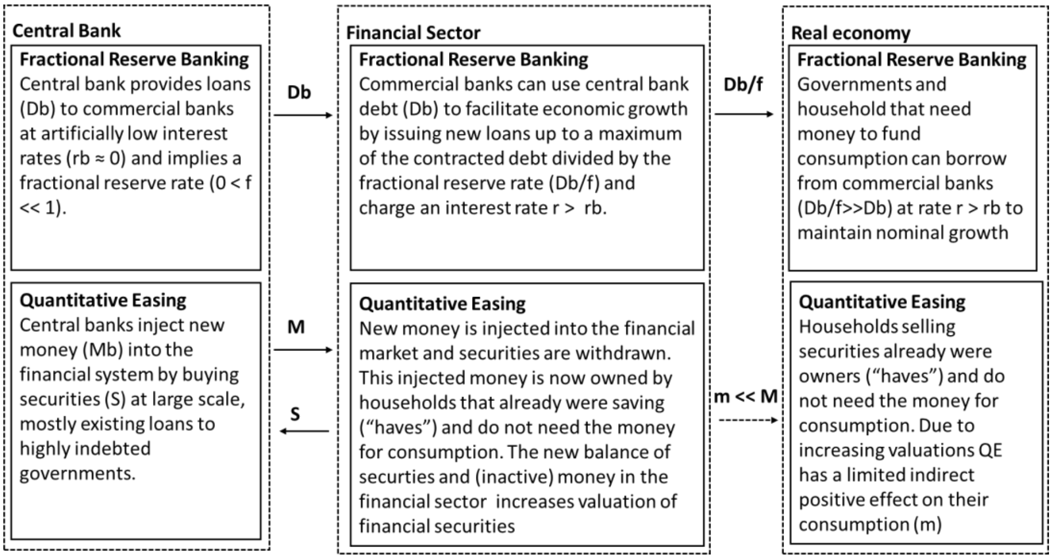

Figure 5.9: Schematic visualisation of mainstream central banks’ policies to stimulate both real economic growth and inflation with fractional reserve banking and quantitative easing

From what I know, stimulating real economic growth is the only reason that virtually all central banks and most mainstream economists and policy makers think a controlled level of inflation is good. As they like to put it; it encourages households to spend their savings now rather than tomorrow and therefore inflation drives economic growth. In my opinion this argument disregards the time value of time. The argument would have been valid if people were to live forever. That way, without inflation it would be worth waiting until goods are cheaper because the period of benefits would remain unchanged. However, in real life people have a chance on dying that generally increases every year. In terms of purchase decisions, they will take this into consideration in order not to spend their lives without the benefit of durable goods. So, when mainstream economists speak about economic growth they in fact mean “volume growth” rather than “productivity increase”. Therefore, to the extent this inflationary policy works real economic growth partly consists of waste in a sense that people buy goods they don’t need yet or replace durable goods too soon, just because they fear higher future prices. This could for example be a television with an incrementally better resolution that replaces an older model that would have been functional for years. Summarising we can say that inflation creates suboptimal volume growth because it focuses on quantity rather than sustainability. However, this is in my opinion the least disturbing factor of inflationary fractional reserve banking.

Now we know that central banks, governments and policymakers of capitalism-based economies throughout the world will do anything to ensure positive GDP growth, preferably at a rate of (close to) 2% inflation above real economic growth, let’s look again at equation (5.23) to see what inflationary fractional reserve banking does to inequality within the economy. Even if all companies annually distribute all their free cashflows as dividends and all shareholders spend all their dividends annually such that the saving rate (1-φ) equals zero every year and all interest *rD* income are reused in the real economy annually, consumers and governments still need to borrow the nominal growth of NDP *(gY')* every year. Therefore, disregarding capital taxes and defaulting debt, the public sector has to (or is "stimulated such that") they borrow a fraction the size of nominal growth of the economy from the private sector every year. Because some households (and most companies) do save, the remaining households and governments will borrow more than nominal growth (i.e. $\varphi<1$). Due to our system of fractional reserve banking all this debt is provided by the private sector. As a result, the debt position of the public sector grows indefinitely. Mainstream economists will argue that this is not a problem, because as long as the debt relative to GDP remains flat the financial system is stable. In section 5.5 Financial Instability we saw that this will most likely be the case in a closed economy with growth and tax parameters in line with the real world. Since the private sector has positive cashflows this will further add to the sustainability of the consolidated debt if you include private debt. However, I believe the ever-growing public debt is a major problem for several reasons.

Firstly, ownership of the private sector (and citizenships which also qualifies as equity) are not equally distributed amongst people nor will it be in the future due to our system of inheritance (see next section). Therefore, inflationary reserve banking creates an ever-increasing liability from people that "have not" (people that depend on labour income) to people that "have" (i.e. owners of equity), even if rich households would not save at all (which they obviously do). This is disregarded in the commonly used definition of capital inequality (equation (5.55)) but included in the definition we use (equation (5.56)). So, although the level of public debt might not increase relative to GDP, the annual increase in debt contributes to ever increasing inequality because it is a net transfer of money from households that save to governments and households that need (or "are stimulated such that they choose") to borrow for their consumption. Obviously, households and governments default every now and then. In my opinion this proves the unsustainability of our financial system rather than it should be considered as a stabilising factor. The supporting spreadsheet monitors the development of β^D over a period of 500 years and allows you to adjust any variable annually if you like.

Secondly, along with ever-increasing debt comes an ever-increasing interest flow. This drives an ever-increasing financial industry that essentially is involved in a zero-sum game. The financial industry does not create real economic value in a sense of productivity increase, it just rearranges exposures to real economic risks and opportunities. Obviously, this slows down real economic growth. In case the average interest rates are higher than nominal economic growth the size of the financial sector relative to the real economy will diverge. Mainstream economy dictates that the risk-free rate should at least be equal to inflation. Therefore, in mature economies with low real economic growth such that the average public interest rates are larger nominal GDP growth the financial system will likely continuously outgrow the underlying real economy to a level that it is no longer sustainable and collapses (like in 2008 for example?). This instability is expressed by for example the simplified budget constraint and can also be modelled in the supporting spreadsheet, both of which do not even include the private interest incomes that could further boost instability.

Thirdly artificial growth of the financial sector is problematic because it draws away labour from the real economy. Because this mostly involves highly educated people like bankers, lawyers, tax specialists and accountants it qualifies as a brain drain from the real economy to the redistributing economy that finance ultimately is. This obviously slows down real economic growth compared to an economy with a small financial sector. Maybe the best example of this untapped opportunity is Jeff Bezos (founder of Amazon) who used to be a hedge fund manager on Wall Street until his early thirties before becoming an entrepreneur. Today, Jeff Bezos is considered one of the best entrepreneurs on the planet. To me this raises the question how much human talent remains untapped in the global financial centres developing and managing "if-you-can't-convince-them-confuse-them-products" instead of using their competencies for the benefit of real economic growth, previously defined as increasing human "quality of life" in chapter 4. I believe the major flaw in today's capitalistic thinking is that is focuses on minimising the size of the government, whereas it should be focused on minimising the size of the financial industry.

Concluding, our current system of inflationary fractional reserve banking comes with four adverse effects. From most adverse to least adverse these are:

- The system inevitably drives increasing inequality due to an ever-increasing debt level from the private sector (owned by the "haves") to the public sector (governments and "have nots") even if all savings would be reinvested into the real economy;
- The system drives artificial growth of the financial sector that yields into a brain drain away from the real economy, which slows down real economic growth;
- The system results into financial crises if the weighted average public interest rate is larger than average nominal economic growth; and
- Inflation stimulates waste due to early purchases and replacements of durable goods;

Because quantitative easing per definition injects money into the financial markets it is either directly or indirectly owned by households that already save or have saved ("haves"). Hence the fraction of this money that is used for consumption is limited. Merely, it drives unnatural valuation increases of financial assets as the balance between inactive money and financial securities shifts towards money.

2. Intergenerational wealth transfer (inheritance)

As far as I am aware in all capitalism-based countries descendants of deceiving people can decline inheritance of parental debt. This way, the aggregated debt position of households stabilises in the long-run. This is because sooner or later households will either (1) pay their debt, (2) default on their debt or (3) die (which then also would be defaulting). This way household debts are ultimately always settled over long periods of time. Unfortunately, governments do not die. Therefore, governmental debt is inherited by new generations. Because (at least since the second world war) most western governments structurally run budget deficits, the public debt gradually increases (mostly also as percentage of GDP). This is a burden, because new citizens pay an increasingly large share of their taxes in the form of interest payments to private holders of debt or could even face repayments of debt that was borrowed by their ancestors. In a closed economy with a non-defaulting government this implies increasing inequality.

The second intergenerational effect that drives inequality is inheritance of privately-owned assets like capital goods, equity, loans and monetary savings. Piketty did a comprehensive analysis on inheritance of capital[103] and concludes that the larger part of all capital in western economies is inherited (as opposed to self-earned) which is expected to grow to 80%-90% in the 21st century in France[104]. In the US inherited capital currently amounts to roughly 50%-80% of all capital[105]. Piketty provides ample data and explanation about how wealth accumulates and increases from one generation to another. Accumulation of capital is probably the main driver of ever-increasing inequality. Off course, inheritance is taxed in most countries. Piketty spends several chapters on tax regimes[106]. It seems that inheritance is progressively taxed with a maximum rate of 35% to 50%[107]. Yet, somehow the fraction of inheritance taxes of government budgets is negligible (well below 1% in case of the Dutch governmental tax income[108] and at least below 2% of the US Federal budget[109]). Therefore, I believe it is fair to say that government debt in western economies is inherited by the public (i.e. every citizen implicitly inherits an equal amount of any burden related to debt her government has) and capital (i.e. equity and savings) are inherited privately (through bloodlines). Now, from historic data and corporate finance theories we know that return on equity on average is roughly 10% annually whereas nominal GDP growth is much less. It is then easy to calculate what level of personal wealth allows for a convenient life from return on capital and still increases your net level of wealth during the course of your live such that your descendants are even richer then you already were when you inherited without one day of working in your life. Just to give an indicative quantitative example of how that could work: Assume a wealthy household requires 10x the average income which in The Netherlands would be roughly EUR 350k annually. The household is exposed to 2% annual

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[103] Piketty Chapter 11, pages 445-508
[104] Piketty page 475 (figure 11.7)
[105] Piketty page 507
[106] Piketty chapters 14, 15
[107] Piketty figure 14.1 page 593
[108] www.rijksbegroting.nl/binaries/pdfs/2/2/1/kst221780.pdf

[109] https://www.thebalance.com/u-s-federal-budget-breakdown-3305789

inflation and owns equity with an annual average return of 10% (in line with historical market average returns in the USA). To maintain the real value of the asset they should reinject (or save) 2% of the value of their assets annually. Therefore, they can spend 8% out of 10% return on equity annually, which would then equal EUR 350k in the first year and increases with 2% annually. This requires a net worth of EUR 4.375 million. So a household that currently owns EUR 4.375 million in a portfolio of assets that yields 10% return annually on average can forever maintain a real consumption level of EUR 350k in an economy that inflates at 2% annually without working. Obviously, when this household has several children these children should all marry children from households in equal situations to prevent dilution of wealth and the families should not grow over generations (≤ 2 children).

A fundamental problem with privately owned equity (which when accounted for at fair value is the larger part of the capital in any western economy) is that it cannot be publicised by increasing inflation. “Printing money” is a well-known trick to reduce governmental debt. However, since inflation always is a result of companies increasing prices, all equity *(E)* in the economy (which is the perfect market portfolio) provides a perfect hedge against inflation. This is especially problematic regarding inheritance tax, that is usually paid (as far as I am aware) in money, not in equity stock. So although heirs might have to pay inheritance tax, they mostly can keep all the equity. This way wealthy people and families are hedged forever against inflation which enables them to grow their wealth ahead of nominal economic growth generation after generation.

On the other hand, we should never forget two things. Firstly, capitalism with it concepts of private ownership and free trade markets is generally accepted as the most successful societal system we are aware of. When it comes to real economic growth, human well-being and happiness, capitalism-based economies are structurally beating other societal systems like dictatorships, socialism and communism. Secondly, we should keep in mind that often a select group of entrepreneurs and entrepreneurial families play a critical role in real economic growth and human well-being. Capitalism enabled these entrepreneurs and entrepreneurial families that consequently gained great personal wealth in doing so. So, it seems the system works well when it comes to creation of wealth but could do better when it comes to the distribution of this wealth. We need to adjust capitalism such that it maintains stimulating people to be entrepreneurial and drive (real) growth and improves the distribution of wealth (over time). To do so, we should consider to adjust both the way we create money and tax inheritance such that the economy converges over generations into a system wherein (1) all new-borns have equal net worth and are exposed to equal opportunities in life and (2) every citizen gets rewarded in life proportional to their personal benefit to common wealth along with a base level of guarantied wealth to protect those citizens who were unlucky to be born with (or accidentally obtained) low intellectual and/or physical capabilities.

6 So, now what?

6.1 What has been happening in the past decades in a nutshell

Emerging economies (either recovering from war or due to established political stability) will build up public debt to finance the growth of the economy. Once the economy is slowing down and approaches a mature state real growth will decline. If the growth is realised by direct foreign investments and financed at the international capital markets (equity and bonds) it will be more difficult for the government in the future to maintain budget control, because there will be an extranational interest and profit flow. During this slowing down the spread between nominal growth and public interest rates decreases to close or below zero. The financial system is now approaching the level that it will become unstable. Because debt is at record levels and growth is slowing down, chances on default increase. This drives increasing interest rates. People get nervous and decrease spending. The impact of these dynamics is (1) increasing economic volatility (i.e. increasing magnitude and frequency of short-term economic cycles) and (2) deflation. Central banks will respond to this by reducing the interest rate and increasing the money supply to stimulate inflation and consumption. From this point onwards, the financial system is decoupled from the real economy, because nominal growth is largely inflationary and access to loans is artificially cheap (i.e. the risk-free rate is below the weighted average annual depreciation of remaining expected lifetime of people and institutions, previously referred to as the time value of time). As a result, the size of the public debt will continue to increase to fund inflation. Interest income will grow faster than real economic growth and the financial sector starts inflating itself. It draws away highly skilled and educated labour from the real economy, which further decreases the pace of innovation and real underlying economic growth. As the size of the financial sector increases an increasing amount of valuable human talent is wasted in the financial centres. Because the savings rate of mature economies is generally positive, there is an increasing amount of cash available for trading securities at the financial markets that was already growing above the pace of the real economy. On top of this central banks often keep injecting fresh money into the financial markets by their policy of quantitative easing. This way, the amount of inactive money (excess cash) in the financial system is further increased. This boosts valuation of capital goods and all kinds of securities like equity, loans, mortgage-backed bonds and alike above their natural value. These capital gains hide the declining growth of the underlying real economy. Not that it matters anyway, because mature economies by nature are the dominant economies on the planet. They now have become system economies with system governments. Central banks will do whatever to takes to maintain financial stability. Whatever the costs, interest rates must be kept lower than inflation. As we know now this even yields negative interest rates, which can never be natural until the day we travel backwards in time.

I believe this has been the situation western economies have been in since at least 2008. Japan maybe much longer. Although the system might be a sort of stable this way, it is not sustainable. I believe the only way out of this is adjusting the system such that it is naturally aligned with the real economy (or what anybody other than economists would call reality). To do so, we should do two things:

- Reject inflationary fractional reserve banking to create financial stability; and
- Adjust inheritance law, practices and systems to establish converging inequality

The alternative is to keep doing what we have always been doing and desperately hope for a different outcome this time. In other words, we could maintain our premise that efficient markets will ensure sustainable financial systems and fairly distribute all wealth between nations and citizens, despite it never happened in the history of mankind.

6.2 Measurement of real economic growth and quality of life

From what I see measurement of real economic growth should not involve money, just human time. To do so we could use the theories described in chapter 4. Measurement of real economic growth of any capitalism-based economy should than not be difficult, just a significant amount of work. To give an idea how measurement of real economic growth might look like, we could separate the economy (or GDP of an economy) in sectors and subsectors like the SBI system in The Netherlands or the NACE classification in Europe. For every (sub-)sector we should then define one volume metric that indicates the output of this segment. For the airline industry this could for example be the total mileage that passengers travelled. For airline builders this could be the flight capacity that was built (for example number of seats of all planes and/or tonnage cargo capacity). For the telco industry this could be total data transmitted, for the producers of communication infrastructure this could be bandwidth installed (and maintained) and for providers of cloud services this could be data stored. For every (sub-)segment the real economic growth would be the sum of the productivity increase and the volume increase. In formula this is:

$$\Delta Q_{i+1,j} = Q_{i+1,j} - Q_{i,j} = q_{i+1,j}L_{i+1,j} - q_{i,j}L_{i,j} = q_{i+1,j}(L_{i+1,j} - L_{i,j}) + (q_{i+1,j} - q_{i,j})L_{i,j} \quad (6.1)$$

In formula (6.1) $Q_{i,j}$ is the total volume output of (e.g. total mileage of the airline industry) of the (sub-)segment *j* of the economy. $L_{i,j}$ is the total amount of labour that was required (for example number of FTE's) and $q_{i,j}$ is the output per unit labour ($Q_{i,j}/ L_{i,j}$). That way, the first term at the right-hand side of the equation ($q_{i+1,j}(L_{i+1,j} - L_{i,j})$) reflects the volume growth of the (sub-)segment *j* in year *i* and the second term ($(q_{i+1,j} - q_{i,j})L_{i,j}$) reflects growth due to productivity increase (i.e. increased output per worker) in this (sub-)segment *j*.

To come to the aggregated real economic growth of the economy we should take the following steps. First we should express the real economic growth of all (sub-)segments *j* into relative values (percentages) as follows:

$$\frac{\Delta Q_{i+1,j}}{Q_{i,j}} = \frac{Q_{i+1,j} - Q_{i,j}}{Q_{i,j}} = \left(\frac{q_{i+1,j}}{q_{i,j}}\right)\left(\frac{L_{i+1,j} - L_{i,j}}{L_{i,j}}\right) + \left(\frac{q_{i+1,j} - q_{i,j}}{q_{i,j}}\right) \equiv \Delta Q_{vol}\%_{i+1,j} + \Delta Q_{prod}\%_{i+1,j} \quad (6.2)$$

The first term of the right-hand side represents the relative (percentage) volume growth of (sub-)segment *j* in year *i+1* and the second terms represents the increase in productivity of (sub-)segment *j* in year *i+1*. The aggregated real economic growth of the economy in year *i+1* ($\Delta Q\%_{1+1}$) is the weighted summation (based on labour input L_{i+1}) of all volume growth ($\Delta Q_{vol}\%_{i+1}$) and productivity increase ($\Delta Q_{prod}\%_{1+1}$) of all (sub-)segments *j* in the economy. In formula this is:

$$\Delta Q\%_{i+1} = \Delta Q_{vol}\%_{i+1} + \Delta Q_{prod}\%_{i+1} = \frac{\sum_{j=1}^{N} L_{i+1,j}\Delta Q_{vol}\%_{i+1,j}}{\sum_{j=1}^{N} L_{i+1,j}} + \frac{\sum_{j=1}^{N} L_{i+1,j}\Delta Q_{prod}\%_{i+1,j}}{\sum_{j=1}^{N} L_{i+1,j}} \quad (6.3)$$

In formula (6.3) the first term at the right-hand side represents the real growth of the economy due to volume growth (increasing consumption) and the second term on the right-hand side represents the real economic growth in year *i+1* due to productivity increase. Now if we recall our definition of quality of life of chapter 3, we can match formula (6.3) to our definition of increase in quality of life to see that real economic growth of an economy can be expressed as (1) increased quality of life due to an increase in consumption and (2) increased quality of life due to productivity increase, which is increase of leisure time. Please note that we disregarded population growth, which could easily be included in the maths.

Now, when we work this out for real economies, we will likely see that there hardly is any increase in leisure time and all real economic growth over the past century[110] has been reinvested into increasing our level of consumption. The obvious reason for this phenomenon is that cash flows and time flows go to different people, which we described in section (5.3.2) as part of our discussion on how microeconomics relates to time accounting. Therefore, the people who can enjoy more leisure time do not have the required increase in income to spend this time pleasantly (as leisure time instead of job seeking time). Therefore, employees will do anything to avoid dismissal or any other form of reduced working hours. As a result there is probably is much slack (i.e. unused labour potential) in the employee base of our companies[111]. To solve this problem I believe both the central banks and our governments should focus their monetary and taxing policies such that (on the long run) free cash flows and free time flows proportionally flow to the same people. If we could develop such a system it would allow average people for the first time in history to choose between (1) increased future consumption or (2) increased leisure time. Obviously in doing so we should not reject all the good

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[110] Among other famous economists John Major Keynes predicted a 15-hour work week by now due to productivity increase (see for example chapter 2 "Een werkweek van 15 uur" of the book "Gratis geld voor iedereen", Rutger Bregman, De Correspondent, 2017)

[111] This indeed is the main conclusion of David Graeber in his book "Bullshit Jobs" (Simon & Schuster, 2018) which shows that roughly 25% of the employees in western economies consider their own job as useless

fundamentals of capitalism such as (1) private ownership, (2) free trade and (3) personal income that is proportional to the public benefits of the individual's efforts (i.e. good entrepreneurs become rich).

In a perfect world we would also distinguish in our definition of real economic growth between real growth of the real economy (previously jointly referred to as B2CG) and real growth of the financial sector (previously jointly referred to as F2CG). This would allow us to monitor growth of the value creation sector (B2CG) and the value distribution sector (F2CG). It would then make sense to develop taxing policies to minimise the size of the financial sector (F2CG), such that we allocate as much labour as we can to the creation of value (B2CG). I believe the major flaw in our current macro-economic and capitalistic thinking is that we are focusing on minimising the government, whereas we should be focussing on minimising the financial sector. This financial sector would include the periphery of finance like lawyers, accountants, tax lawyers and alike which all are specialists involved in value distribution more than value creation and which all come at much higher hourly rates than most specialists in the "real" (i.e. value creating) economy like professors, teachers, doctors, researchers, pilots, engineers, factory operators and alike. CEO's have high incomes, but probably largely because they are incentivised with equity schemes.

The next two paragraphs subsequently describe such central banking and taxing policies, which we will refer to as sustainable central banking and sustainable tax regimes respectively.

6.3 Sustainable central banking

From what I see there are two flaws in our current financial system that make it fundamentally unstable. Firstly, our current financial system (due to fractional reserve banking and the principle of balance in financial accounting) makes it a **zero-sum game** that we use to model the **value creation process** that is going on in the real economy. In other words, we must include a source of money in our financial system that flows to real consumers (households and governments) and as such grows the money supply in the real economy in line with the real growth and is not booked against new debt to financial institutions. Today, all real economic growth is captured by the private sector by issuing debt against money to fund real economic growth.

Secondly, there exists a drain from the money supply in the real economy the size of the aggregated free cash flow of the private sector minus the fraction that is distributed as dividends ánd reused for consumption by households (previously defined as $\Theta Y'_i$). This drain of money could either be directly reinjected into the real economy or we should accept deflation.

Today, western central banks (a.o. Federal Reserve, European Central Bank, Bank of England) have a policy to aim for both real growth and inflation, which they do by stimulating companies, governments and households to borrow money. If this no longer works central banks inject money into the aggregated financial markets to stimulate inflation, whereas I believe it would make more sense to inject money directly into the real economy. The long-term result of common central banking policies is (1) an ever-increasing amount of public debt and (2) an ever-increasing amount of inactive money in the financial markets that drives values of securities, but will never find its way back into the real economy.

Based on (1) the lack of a money source in the real economy and (2) the drain of money due to savings I believe central banks (jointly with governments) should do the exact opposite of what they have been doing the past decades in order to maintain a sustainable financial system. They might want to consider looking for ways to annually withdraw all additions to savings (new inactive money) from the financial markets and directly inject an equal amount into the real economy added with an amount to cover real economic growth. This way central banks would ensure that (1) the money supply in the real economy grows in line with the size of the real economy, (2) public debt and interest payments do not grow exponentially and (3) there is no abundance of inactive money in the financial markets that artificially boosts valuation of securities. The private sector (B2CG) has a positive cashflow (roughly 25% of GDP) hence generates enough cash to finance real growth of roughly 1-3% of GDP). Such a central banking policy could look like the following.

The first step would be to define the most recent real economic growth g_r of the economy like we just did in the former section (equation 6.3). This is the amount of money (relative to $NDP=Y'$) that should be created ("printed like helicopter money") and deposited pro rate parte their consumption of respectively G and C on the bank accounts of the governments and households. This would be a gift (as opposed to a loan) which prevents deflation and allows the public sector to maintain their consumption in line with real economic growth (assuming wages growing in line with real economic growth) without borrowing. Secondly, we should bear in mind that the private sector has positive annual free cash flows (see also table 5.4). Although part of these cashflows will be reinjected into the real economy by household consumption out of dividends and by dividend taxes, it is evident that rich households (and a lot of companies) jointly annually save part of the aggregated free cashflows. The central bank should measure the joint savings of all companies and households (and potentially governments) that add up to what we defined earlier as *Θ=1-*φ by using the exact budget constraint (formula (4.23).

$$\frac{\Delta D_{i+1}}{Y'_i} = g + (1-\varphi) + \left[r\frac{D_i}{Y'_i} - \varphi r \frac{D_{i-1}}{Y'_i} \right] \quad (5.23)$$

To avoid unnaturally high asset and security prices, the central bank should annually remove an amount of $\Theta Y'_i$ from the aggregated financial markets by issuing bonds. If markets are efficient, the interest rate on these bonds with zero chance on default apparently is the "time value of time" perception of the people and organisations in the economy. This way, an equal amount of *ΘY'* is withdrawn from the financial markets such that the total inactive money supply maintains unchanged. Arguably, this amount should grow in line with the consolidated profits or cashflows of the private sector to maintain a balanced amount of liquidity in the financial markets. This policy should yield into an annual growth of the amount of inactive money in the aggregated financial markets by the annual increase of the consolidated NOPLAT of the private sector divided by the cost of capital (WACC) minus expected annual average future growth. This way, the amount of money in the aggregated financial markets remains stable relative to the value of all assets (disregarding fixed-income bonds).

Now, the real economy still suffers from a shortage of *Θ* times *Y'* that is withdrawn from the money supply. To prevent either (1) deflation and/or (2) an ever-increasing unnatural public debt position, the central bank of a closed economy with no population growth should annually reinject a fraction *Θ* times *Y'* into the real economy. Preferably this amount of money is created by pro rate parte free deposits to the government(s) and households (or citizens). In case of an open economy with a trading surplus this fraction should be less (because part of the free cashflows are obtained by international trading) and in case of an economy with a trading deficit this fraction should be higher. If currency exchange markets are efficient this results in natural devaluation of the currency of the economy with a trading deficit compared to currencies of exporting countries, which reflects the difference in values of human time between countries. If the real growth would be negative, the central bank should in theory withdraw money from citizens and governments. It might be better though to account for accumulated negative growth and deduct this from future deposits later on (it would be like the opposite of tax-deductible losses).

Real economic growth is typically below 1% of *NDP (Y')* in mature economies and the saving quote roughly 5%[112]. To maintain a money supply of a closed mature economy that grows in line with the size of the real economy the central bank should then roughly issue 5% of NDP *(Y)'* of risk-free bonds and deposit this money on bank accounts of citizens and government(s) along with another ~1% to cover real economic growth. As an indicative example we could use the USA GDP and population to estimate the amount ($g_r Y'$) that would be directly injected into the real economy annually and (partly) withdrawn from the financial markets back into the real economy *(ΘY'* less an increase to compensate for natural capital gains*).* USA LE 2019 GDP amounts to USD 21.5 trillion[113], which typically breaks down into C≈60%, C≈30% and I≈10% (ignoring international trading). Therefore, *NDP (Y'=Y-I)* amounts to USD 19.35 trillion, of which government consumption *(G)* in our example amounts to USD 6.45 and consumption *(C)* is USD 12.9 trillion. If real growth amounts to 1% on average the central bank should print 1% of NDP which equals USD 194 billion. A fraction G/NDP which amounts to USD 65 billion would be deposited at the bank account of the government (1% of their annual spending). The remaining amount would be deposited on every citizen's (or households) bank account, which would also be on average 1% of consumer spending. In case of our example this gift from the central bank to every citizen would

112 https://data.oecd.org/hha/household-savings.htm
113 https://en.wikipedia.org/wiki/Economy_of_the_United_States

be roughly USD 400 per citizen annually. Reinjecting annual savings of *ϴY'* assuming *ϴ* roughly equals 5% of NDP would implicate that the central bank (Federal Reserve) would annually issue USD 968 billion (or less to compensate natural capital gains) of risk-free rate loans into the aggregated financial markets and redistribute this amount of money to both the US government (5% of their spending) and households/ citizens (roughly USD 2.000 per citizen, ignoring international trade effects). These measures (especially reinjecting *ϴY'*) may seem quite unconventional and I am not saying central banks should radically change their current policies. Nonetheless reinjecting money into the real economy in the form of gifts as opposed to loans is the only measure I can think of to obtain (1) a sustainable financial system and (2) reduce the pace of ever-increasing inequality.

If central banks adopt policies in line with the above, I believe other central banking practices should remain unchanged. That is (1) independency from the government and (2) fractional reserve banking to maintain a natural level of debt (i.e. elderly and rich people providing loans to young people and entrepreneurs through a commercial system of banks) and a natural level of interest (i.e. an interest rate that reflects the time value of time of the lender and the probability weighted risk on default of the borrower).

6.4 Sustainable tax regimes

When it comes to redistributing wealth by taxing, raising income taxes like corporate income taxes and dividend taxes is likely not going to work in most situations. To see this, please recall equation (5.46).

$$\left[\frac{\Delta D_{i+1}^{G}}{Y_i}\right]_{\frac{\Delta D_{i+1}}{Y'_i}} = [\gamma g]_g + \left[\gamma - \tau_{CIT} + \tau_L(1-\alpha) + \tau_{DIV}\left(\frac{DIV}{FCF}\right)(\alpha - \tau_{CIT} - s)\right]_{(1-\varphi)}$$

$$+\left[\left(\frac{r^G D_i^G}{Y_i}\right) - \left(\tau_{CIT}\left(\frac{r_i D_i}{Y_i}\right) + \tau_L(1-\alpha)\left(\frac{r_i D_i}{Y_i}\right) + \tau_{DIV}\left(\frac{DIV}{FCF}\right)(\alpha - \tau_{CIT})\left(\frac{r_i D_i}{Y_i}\right)\right)\right]_{r_{net}D_i/Y_i} \qquad (5.46)$$

Generally speaking, governments that could easily do this are countries that (1) have a large real economy per capita (i.e. an international trading surplus) and/or (2) a large (international) financial industry. In these countries inequality between citizens is mostly not urgent. It is the poor countries and countries with a trading deficit that suffer most from inequality. And raising income taxes and dividend taxes would further incentivise the private sector to move away from them in today's competing fiscal environment between governments. Nonetheless, for those countries that both suffer from increasing inequality and have a mature private sector raising income taxes could help. These countries would for example be The United States or The United Kingdom.

Also governments could consider to imply a higher corporate income tax rate to the financial sector than to real economy companies (B2CG) in an attempt to reduce the size of the financial sector.

Nevertheless, the preferable way to fight increasing inequality in my opinion would be to imply capital taxes, inheritance tax most of all. This would maintain the competitive business environment that capitalism-based economies typically provide for companies. Obviously, inheritance tax should be (i) progressive in order not to take savings from average people and (2) organised such that it would be impossible to live from income from inherited capital and pass on more capital (adjusted for inflation) to the next generation. In other words, the annualised inheritance tax rate should be higher than the average spread between return on capital and nominal GDP growth.

Preferably inheritance tax should be paid in kind (equity shares as opposed to money) so the tax is protected against inflation. These equity stakes (arguably the legal voting rights stay with the families to respect and benefit from their entrepreneurial talents) could then be kept in a specific purpose vehicle managed by the central bank (independently from governments). This would gradually grow into a large state fund that generates dividends to which all (world) citizens are equally entitled. Dividends can be used to (1) settle against issued risk-free loans to remove money permanently out of the system, (2) pay interest obligations of the central banks, (3) inject money into the real economy to cover real economic growth or (4) use as societal dividend (i.e. basic income).

6.5 The American Dream

If you practiced a bit with the spreadsheet (sheet 5.6 inequality) you will have learned that is impossible for our current leadership (governments and central banks) to maintain a sustainable financial system. If you don't like spreadsheet modelling you could also invite a few friends to play a game of Monopoly and see if you can end up in a sustainable situation wherein both money and hotels remain equally distributed amongst the players. In other words, a situation wherein the game never ends. The more likely outcome is that one player owns virtually all hotels and keeps providing loans to the other players because she does not want the game to end. Meanwhile, all the other players are increasingly frustrated and tempted to throw the game board from the table. This happens even when we start with (1) equal opportunities, (2) equal net worth (20k of starting capital) and (3) a basic income for every player of 20k for every time we pass "start". Imagine playing a game of Monopoly wherein a few players start with both money and a few hotels and others have nothing. This would be the more accurate representation of our current global economy.

Now I believe that if we respect that money is the equivalent of human time, we can obtain natural capitalism which is both sustainable (i.e. fair and stable) and stimulates entrepreneurship (i.e. encourages individuals to drive real economic growth). In today's global economy some people are born with a virtually uncapped claim on other people's time for the benefit of their own consumption and well-being whereas many other people will have to give virtually all the time they have in their lives to other people to prevent their families from starving.

Once you think you know a thing or two about valuation and distribution it is easy to imagine a perfect capitalism state. Let's refer to this place as "**Capitaltopia**". It's a place where every newborn soul has equal opportunities and starts with the same net worth. This net worth is all equity (which is accounted for as all the capital in the economy) that was a result of value created by former generations (people who already deceased) divided by all the people alive. Like when we play Monopoly (please ignore the name of the game as it now detonates with the context); we all start from the same place with the same starting capital. We would consider any other way to start the game unfair.

Obviously, there are no countries in Capitaltopia since citizenships are equity. There is only one global democratically elected government and an independent democratically elected central bank. Individual incomes of people should be proportional to their individual contribution to the economy based on the principle that "you get what you give". Once people die their net worth is (gradually over generations) inherited by the community (e.g. by a publicly owned fund or the central bank). This way, the system properly supports the entrepreneurial people that have been proven to be so important for our continuous development and growth. On the other hand, the system also provides for a basic level of living and comfort for those people who are less adventurous. The latter group probably is most of mankind, which jointly adds up to our labour force (save aside a small portion of disabled people that the community takes care of). A system based on these principles would give the people a balanced choice in their lives between leisure time and consumption and it appropriately rewards the exceptional souls that disrupt and grow our civilisation to the next level for the benefit of future generations.

As a non-American it might not be appropriate to have an opinion about the American Dream, but I think this is it.

7 Appendices

7.1 Value of an investing economy with population growth in discrete time

We will now consider an economy that invests a fraction *s* of its annual aggregated production Y_i in capital goods for any given year *i* with $i > 0$. Because of the annual investment sY_i into capital goods every year the economy is expected to grow with a fraction gY_{i+1} in the year thereafter for any given year *i* with $i > 0$. The capital goods are expected to devaluate with a depreciation rate δ such that the expected aggregated annual productivity decrease of the capital goods amounts to $Y_i /(1+ \delta)$. The population of this economy grows at constant rate with a fraction *n* every year from year *1=2* onwards. Since the people invest sY_i into capital goods the aggregate consumption in any given year *i* the aggregated consumption at any given year will be $C_i=(1-s)Y_i$. Therefore, the present value of the aggregated consumption of year *1* can be written as formula (4.17):

$$C_1 = (1-s)Y_1 = (1-s)\frac{Y_0}{(1+\delta)} = \frac{(1-s)Y_0}{(1+\delta_0+\delta_T)} \tag{4.17}$$

From year 2 onwards, the aggregated production is expected to grow driven by both the productivity growth *g* and population growth *n* and the present value of aggregated consumption in year 2 can be written as formula (6.1)

$$C_2 = (1-s)Y_2 = (1-s)Y_1\frac{(1+g)}{(1+\delta)} = (1-s)Y_0\frac{(1+g)(1+n)}{(1+\delta)^2}, with\ \delta = \delta_0 + \delta_T \tag{7.1}$$

This is equal to equation (4.18) multiplied by *(1+n)* to account for population growth. For any given year *i* afterwards the aggregated annual production is provided by

$$C_i = (1-s)Y_i = \frac{(1-s)Y_0}{(1+\delta)}\left(\frac{(1+g)(1+n)}{(1+\delta)}\right)^i \tag{7.2}$$

Therefore the value $V_{s,n}$ of an economy with population growth *n,* productivity growth *g* and depreciation rate *1/(1+δ)* is expressed by (7.3):

$$V_{s,n} = \sum_{i=1}^{\infty}\frac{(1-s)Y_0}{(1+\delta)}\left(\frac{(1+g)(1+n)}{(1+\delta)}\right)^i = \frac{(1-s)Y_0}{(1+\delta)}(1+x+x^2+\cdots+x^\infty), with\ x = \left(\frac{(1+g)(1+n)}{(1+\delta)}\right) \tag{7.3}$$

Provided that $x < 1$ we can replace the polynomial $(1+x+x^2+\ldots+ x^\infty)$ by $1/(1-x)$:

$$V_{s,n} = \frac{(1-s)Y_0}{(1+\delta)}\left(\frac{1}{1-x}\right), with\ x = \left(\frac{(1+g)(1+n)}{(1+\delta)}\right) \tag{7.4}$$

Replacing x yields:

$$V_{s,n} = \frac{(1-s)Y_0}{(1+\delta)}\left(\frac{1}{1-\frac{(1+g)(1+n)}{(1+\delta)}}\right) = \frac{(1-s)Y_0}{(1+\delta)}\left(\frac{1}{\frac{(1+\delta)}{(1+\delta)}-\frac{(1+g)(1+n)}{(1+\delta)}}\right) = \frac{(1-s)Y_0}{(1+\delta)}\left(\frac{1}{\frac{(1+\delta)-(1+g)(1+n)}{(1+\delta)}}\right)$$

$$V_{s,n} = \frac{(1-s)Y_0}{(1+\delta)}\left(\frac{1}{\frac{1+\delta-1-g-n-ng)}{(1+\delta)}}\right) = \frac{(1-s)Y_0}{(1+\delta)}\left(\frac{1}{\frac{\delta-g-n-ng)}{(1+\delta)}}\right) \tag{7.5}$$

If we multiply both the denominator and the numerator of the second term by *(1+ δ) and* subsequently divide both the denominator and the numerator of right side of the formula by *(1+ δ)* we get:

$$V_{s,n} = \frac{(1-s)Y_0}{(1+\delta)}\left(\frac{(1+\delta)}{\delta-g-n-ng)}\right) = \frac{(1-s)Y_0}{(\delta-g-n-ng)} \tag{7.6}$$

If the growth *n* and *g* are much smaller than 1, which seems a reasonable assumption in case of mature economies[114], than the term $n*g$ is much smaller than *n* and *g* and can be ignored. In formula:

[114] Piketty (page 115-117 estimates long-term real growth of the mature economies like Western-European countries and the USA to be a maximum of 1.5% per year and the long-term annual population growth *n* of these countries to be around 1%. Then $n*g$ is 0,015% which is negligible compared to *n=1% and g=1,5%.*

$$V_{s,n} = \frac{(1-s)Y_0}{(\delta-g-n-ng)} \approx \frac{(1-s)Y_0}{(\delta-g-n)}, if\ n << 1\ and\ g << 1 \quad (7.7)$$

7.2 Value of an investing economy with population growth in continuous time

In this section we will derive a formula like (7.7) that describes the value of an investing economy with population growth using a continuous-time model. To do so, let us assume that *Y(t)* represents the total aggregated production of an economy that invests a fraction *s* of its production in capital goods that drives an expected constant productivity growth of *g* and has a population that growths constantly over time with *n.* Due to the limited lifespan of capital goods in the economy, the aggregated productivity depreciates at a constant rate δ_0.The depreciation rate of human time is δ_T.

Like the neo-classical models like the Solow-Swan model[115] we assume that the available aggregated labour *Y(t)* can be expressed as the multiple of the amount of labour available *L(t)* multiplied by a term *A(t)* that describes the evolution of the productivity of labour expressed per unit labour. In classical models *A(t)* is often referred to as the "Labour-augmenting technology" or "knowledge". The aggregated production *Y(t)* is the aggregated available labour *L(t)* multiplied by the productivity function *A(t)* and can be written as formula's (7.8), (7.9) and (7.10),

$$Y(t) = A(t)L(t) \quad (7.8)$$

with:

$$L(t) = L(0)e^{nt} \quad (7.9)$$

and

$$A(t) = A(0)e^{gt} \quad (7.10)$$

L(0) is the amount of labour available at *t=0* (including the amount that is in invested in capital goods should the people choose to do so) and *A(0)* denotes the level of productivity per unit labour at time *t=0.*

The aggregated consumption is the aggregated production *Y(t)* minus the portion *sY(t)* that is invested in capital goods. In formula we write:

$$C(t) = Y(t) - sY(t) = (1-s)Y(t) \quad (7.11)$$

Now we can write *Y(t)* as the product of *L(t)* and *A(t)* like *Y(t)=L(t)A(t).* To include the depreciation rates δ_0 of capital goods within the economy and δ_T of human time that respectively drive productivity loss of the aggregated production and relates the value of future time to present time we define a function *D(t)* similar to *L(t)* and *A(t)* as follows

$$D(t) = D(0)e^{-(\delta_T+\delta_0)t} \quad (7.12)$$

Please note the minus sign ("-") in the exponential term $e^{-\delta t}$ to represent the fact that depreciation drives negative growth of aggregated productivity. Including *D(t)* in *Y(t)* yields (7.13) that represents that represent the present value of the aggregated consumption *C(t)* at time *t*.

$$C(t) = (1-s)Y(t) = (1-s)D(t)A(t)L(t) = (1-s)D(0)A(0)L(0)e^{-(\delta_T+\delta_0-g-n)t} \quad (7.13)$$

Now we define *Y(0)* as the aggregated available production at *t=0,* similar to its meaning in our discrete time model, such that:

$$Y(t=0) = Y(0) = D(0)A(0)L(0) \quad (7.14)$$

We use (7.14) to replace *D(0)A(0)L(0)* by *Y(0)* into formula (7.13) to denote the value at present time of the aggregated consumption of the economy *C(t)* as follows.

[115] https://en.wikipedia.org/wiki/Solow%E2%80%93Swan_model

$$C(t) = (1-s)Y(0)e^{-(\delta_T+\delta_0-g-n)t} \tag{7.15}$$

To obtain the value $V_{s,n}$ of the economy we need to integrate the aggregated consumption function *C(t)* from present time (*t=0) to* infinity (*t=∞).* If formula this can be written as (7.16).

$$V_{s,n} = \int_0^{\infty} C(t)dt \tag{7.16}$$

Solving this for and economy that has an aggregated consumption function *C(t)* as given in (7.15) results in (7.17).

$$V_{s,n} = \int_0^{\infty} C(t)dt = \int_0^{\infty}(1-s)Y(0)e^{-(\delta_T+\delta_0-g-n)t}dt = \left[\frac{-(1-s)Y(0)e^{-(\delta_T+\delta_0-g-n)t}}{(\delta_T+\delta_0-g-n)}\right]_0^{\infty} \tag{7.17}$$

This can be further solved as follows.

$$V_{s,n} = \left[\frac{-(1-s)Y(0)e^{-(\delta_T+\delta_0-g-n)t}}{(\delta_T+\delta_0-g-n)}\right]_0^{\infty} = \left(\frac{-(1-s)Y(0)}{(\delta_T+\delta_0-g-n)}\right)[0-1] = \frac{(1-s)Y(0)}{(\delta_T+\delta_0-g-n)} \tag{7.18}$$

Please note that this formula is exactly equal to formula (7.7) we derived in appendix 7.1 in a discrete time model and equals formula 4.24 in chapter 4 if we assume no population growth (*n=0).*

It is easy to see that if we define more components $B(0)e^{-\beta t}$ to describe the function *Y(t)= D(t)A(t)L(t)B(t)* the result will be that *β* will be included in the numerator of equation (7.18) like $V_{s,n}=(1-s)Y(0)/(\delta_0+\delta_T-g-n-\beta)$.

BACK PAGE

TIME TO VALUE

Production - Like classical economists we believe that labour (useful human time) is the only source of value and therefore we consider labour the only production factor in the real economy. This approach excludes "capital" as a separate production factor because it refers to "capital goods", which equal to consumption goods are gradually created through the value chain exclusively by labour that transfers energy and matter (i.e. natural resources) that are freely available to humanity into useful goods. This view enables us to denote the aggregated production costs of all capital goods, consumption goods and (obviously) services in a closed economy in terms of human time. The value of natural resources then is the value they carry in a final useful state (either a consumption good or a capital good) minus the investment of human time required to get there.

Consumption – Consumption of value occurs in two ways. Firstly, people consume goods (consumption of stored labour from the past) and services (instant consumption of labour). Secondly, people consume the opportunity costs of their own labour potential whenever they are not working (consuming leisure time).

Time valuation and the time value of time - In analogy with the "time value of money" we introduce the "time value of time" that states that present time is more valuable for humans than future time, which we use to discount future human time to derive the net present value of a closed economy in terms of human time instead of monetary value. It appears that there is an optimal fraction of labour that people can invest in the creation and maintenance of capital goods that maximizes the value of a closed economy.

Time accounting - Finally we show how to account for human time by introducing the aggregated time statements of a closed economy (in analogy with the financial statements of a company).

Time dividend - Time accounting and time valuation show that if the people in a closed economy invest part of their labour capacity in the creation of capital goods they will increase their future labour productivity which they can use to improve their future "quality of life", either by reinvesting time in more future consumption of goods and services (real economic growth) or by consuming more future leisure time (time dividend).

MONEY TO SHARE

Aggregated time flows and the natural dynamics of economies - Finance is humanity's big ledger with a purpose to account for all transactions between people in the real economy to keep track of all claims they have on each other and on (semi-finished) goods, raw materials and natural resources. Since we can now denote both the costs and value of all goods and services in terms of human time we can see that money in the financial system is the equivalent of human time in the real economy. All transactions in the economy are an exchange between human time and money and jointly generate both aggregated cash flows and aggregated (human) time flows that move in opposite directions.

We relate time valuation and time accounting to asset pricing, micro- and macroeconomics to assess how well our current economic theories and practices align with what we believe are the natural dynamics of economies.

Valuation - Based on these analyses we argue that the "risk-free rate" that is commonly used in asset pricing reflects the weighted average of individual "time values of time" of investors and is not the return investors demand on a riskless asset. The individuality of the risk-free rate is not included in common asset pricing models like the Capital Asset Pricing Model (CAPM).

Accounting – Like stock flow consistent modelling we use the principles of bookkeeping and financial accounting to consolidate the financial statements of all companies and financial institutions (jointly referred to as the private sector) in a fictive closed economy which we use to derive the budget constraint of all households and governments (jointly referred to as the public sector). This way, we reveal among other things that the way most western economies create money (fractional reserve banking) is a fundamental driver of financial instability and ever-increasing inequality. We provide some guidance on how we might be able to fix this.